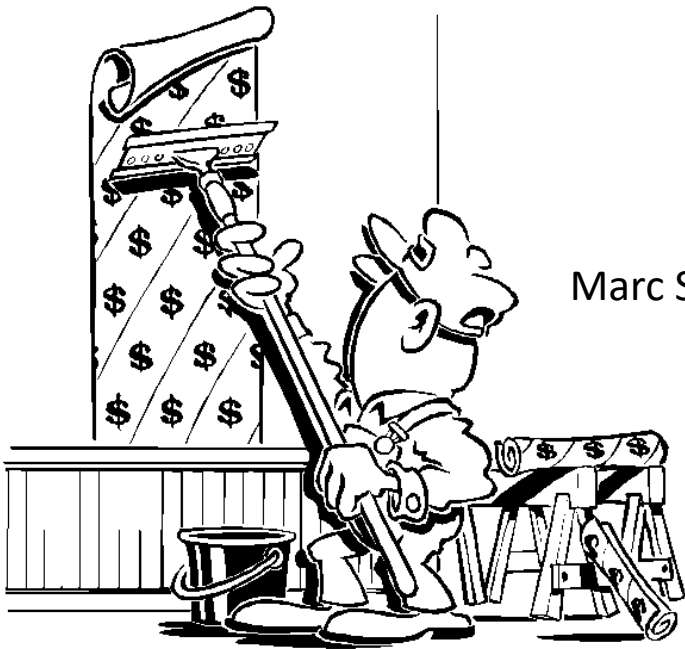


## Lecture #10

# Texture Mapping

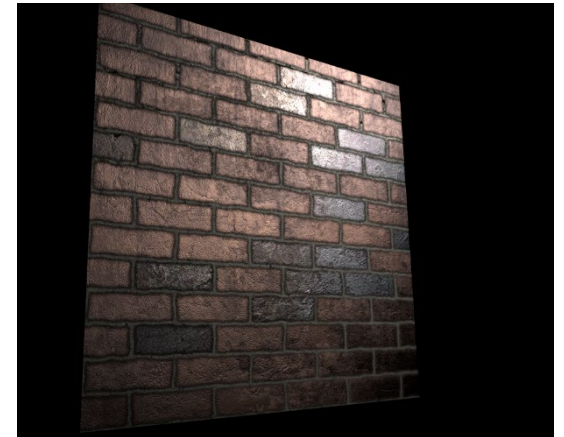
Computer Graphics  
Winter Term 2020/21

Marc Stamminger / Roberto Grosso



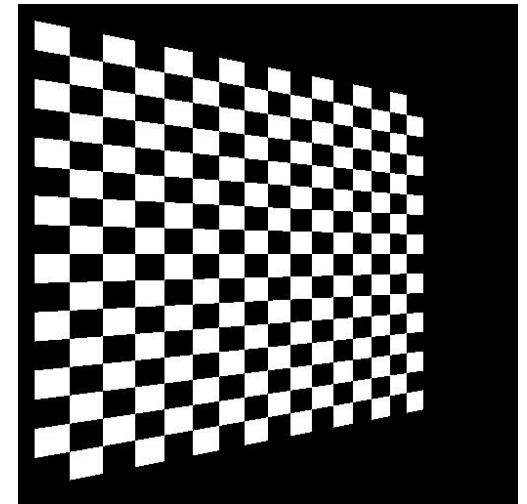
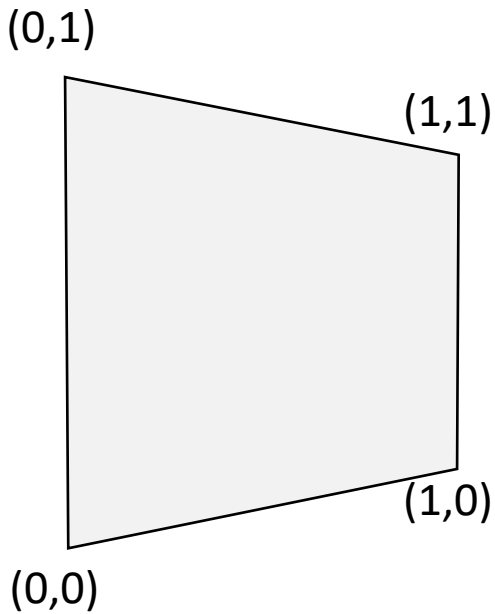
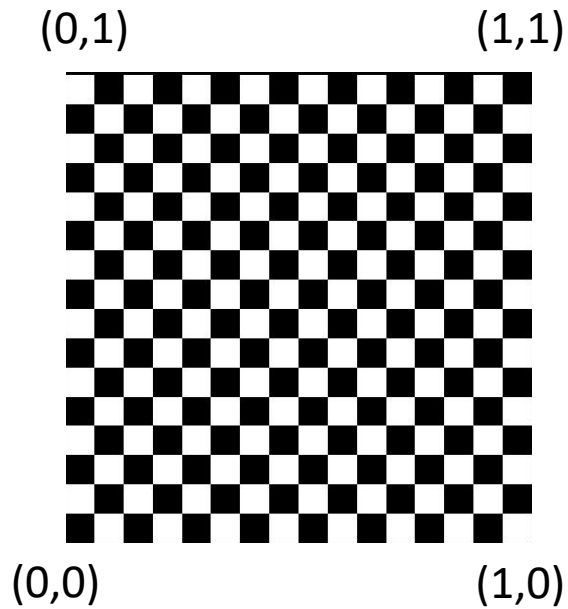
# Texture Mapping

- so far: detail through polygons & materials
- example: (large) brick wall
  - many polygons & materials needed for bricks  
→ inefficient for memory and processing
- alternative: **Textures**  
introduced by Ed Catmull (1974)  
extended by Jim Blinn (1976)

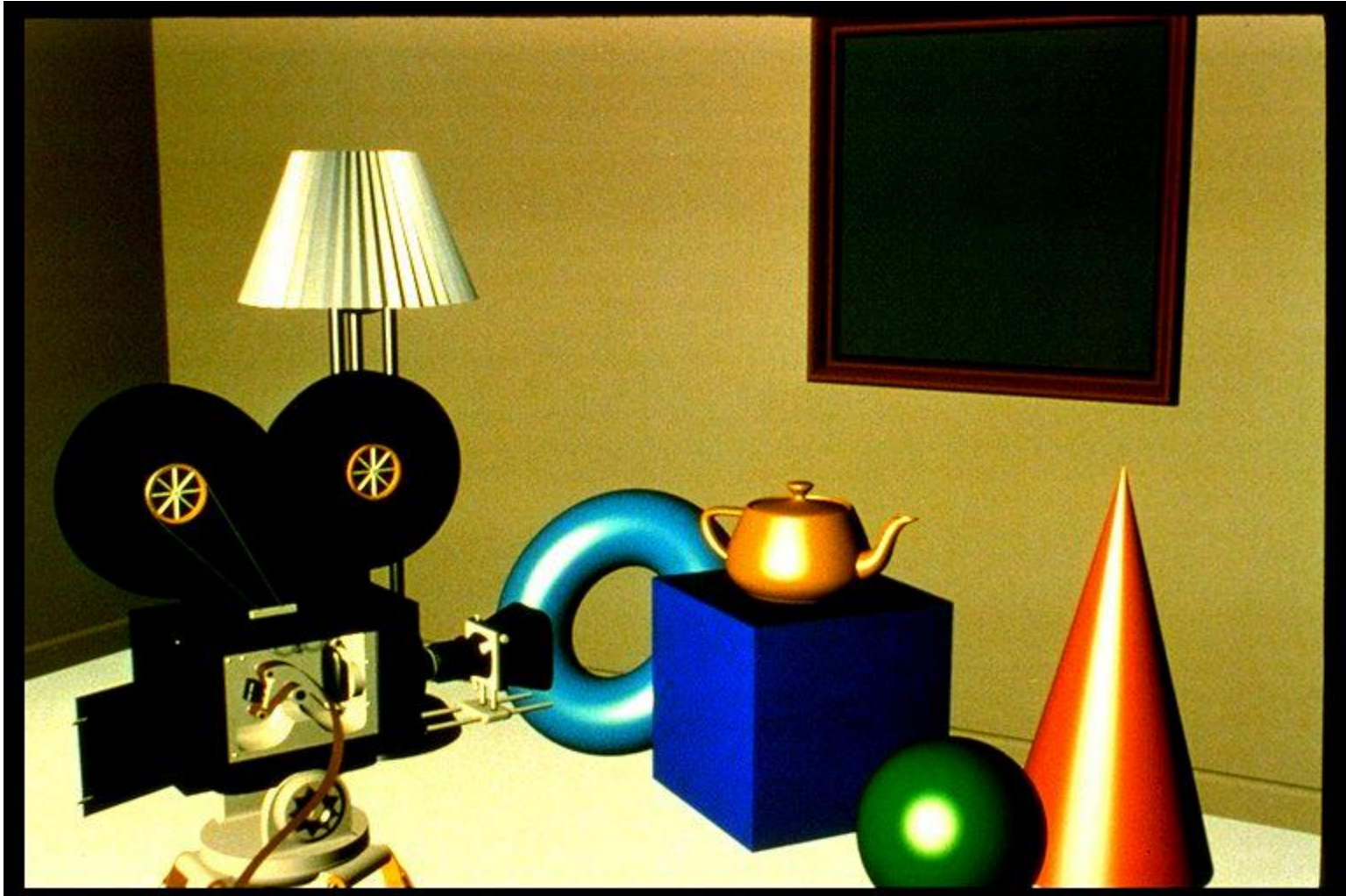


# Texture Mapping

• Texture + Quad = Image



# Texture Mapping



Foley, van Dam, Feiner, Hughes

# Texture Mapping



Foley, van Dam, Feiner, Hughes

# Texture Mapping

- What are textures or texture maps?
  - Functions or images that change the appearance of an object, typically its color
    - Coarse geometry (i.e. fast rendering), fine texture (i.e. fine visual detail)
  - Great performance gain compared to using huge triangle meshes with different materials
  - Can be 1D
    - heat map: maps the “temperature” of an object to color(cold=blue, warm=red)
  - or 2D
    - images to mapped onto the object like wall paper
  - or 3D
    - volumetric objects such as clouds
    - or solid objects such as wood
- **for now, we only look at 2D textures**

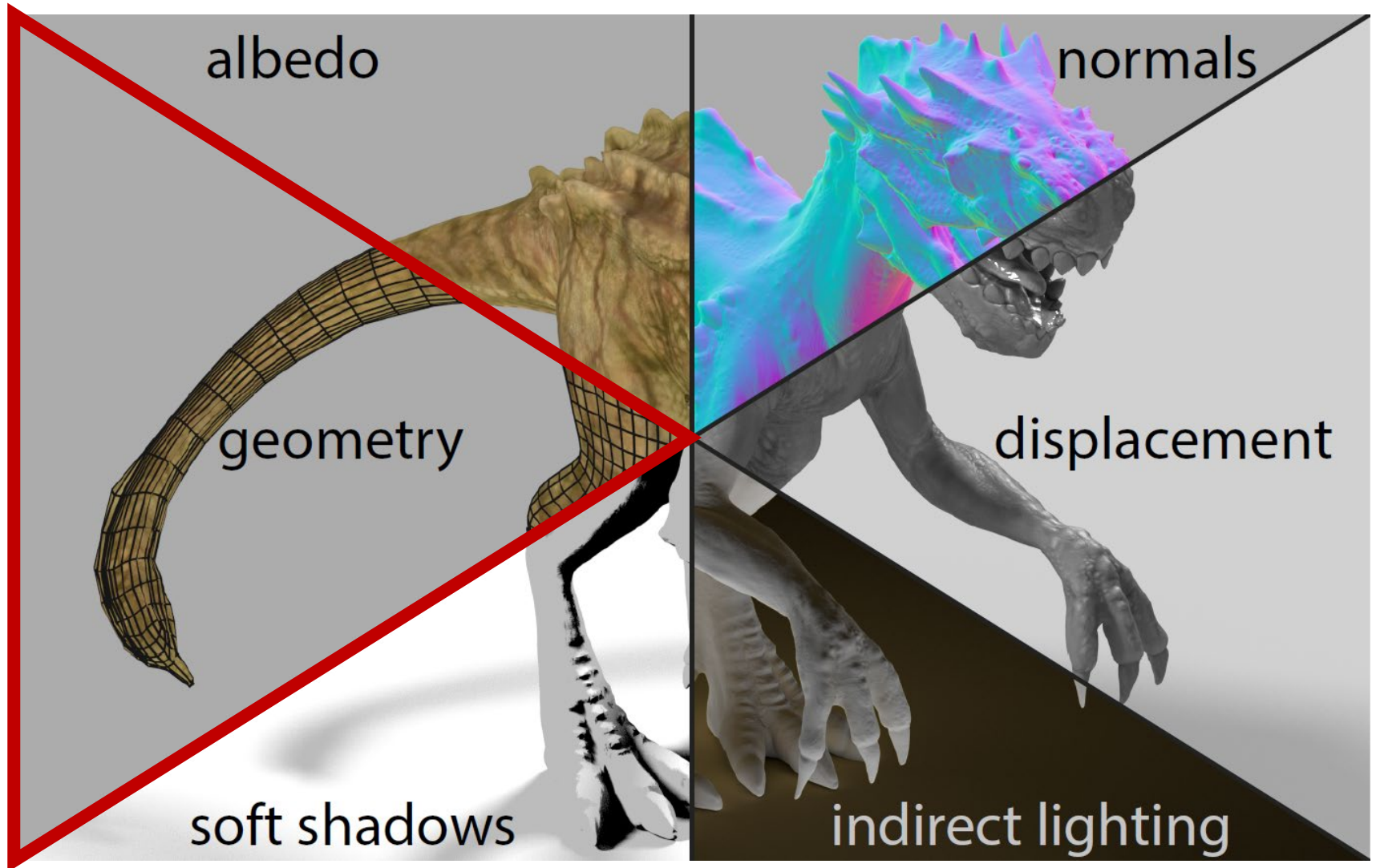


# Texture Mapping

- Textures usually contain color, e.g. the diffuse component of the Phong model
- But they can also contain specular color, ambient color or other material parameters
- And even much more!

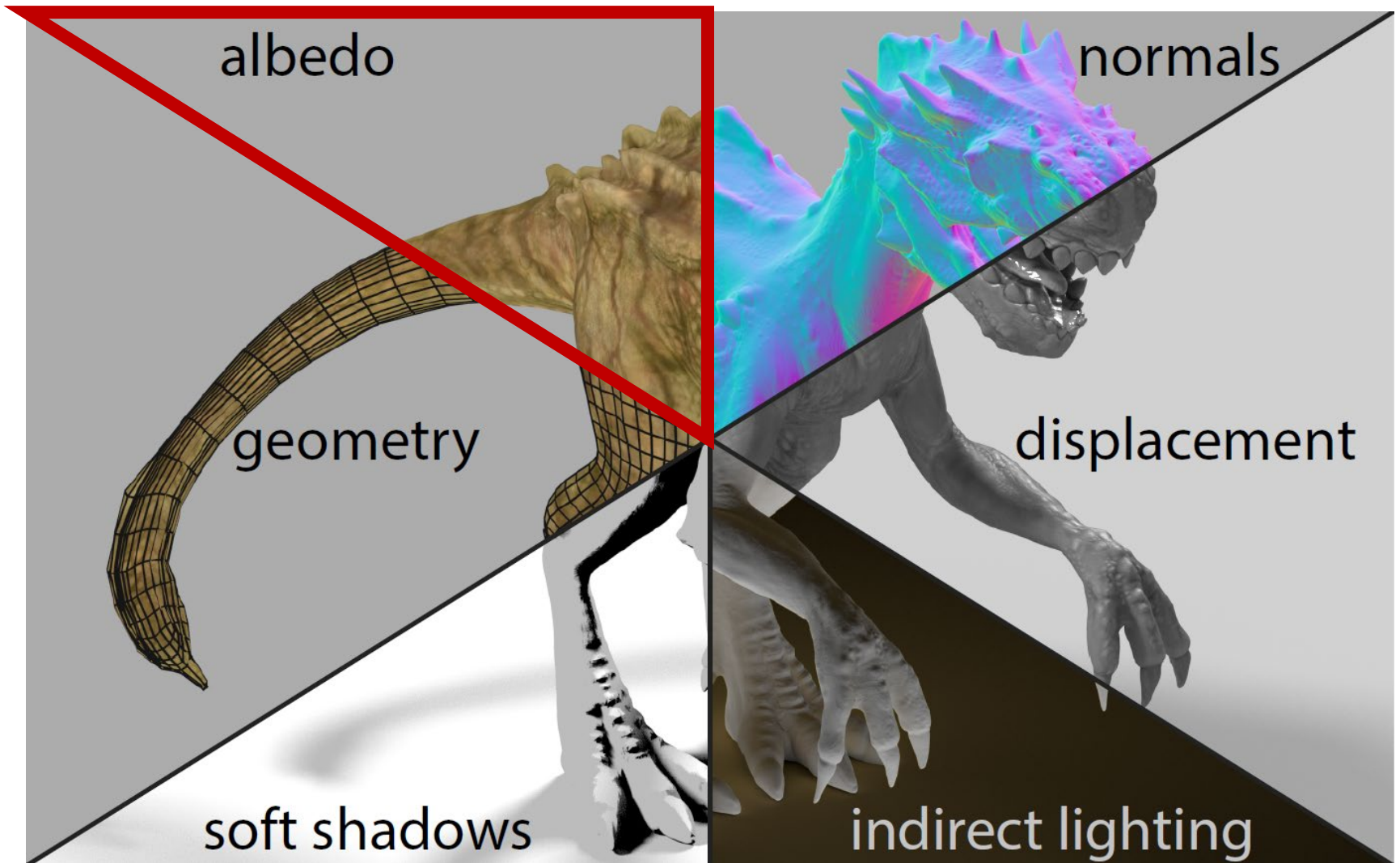


# Texture Mapping - Introduction

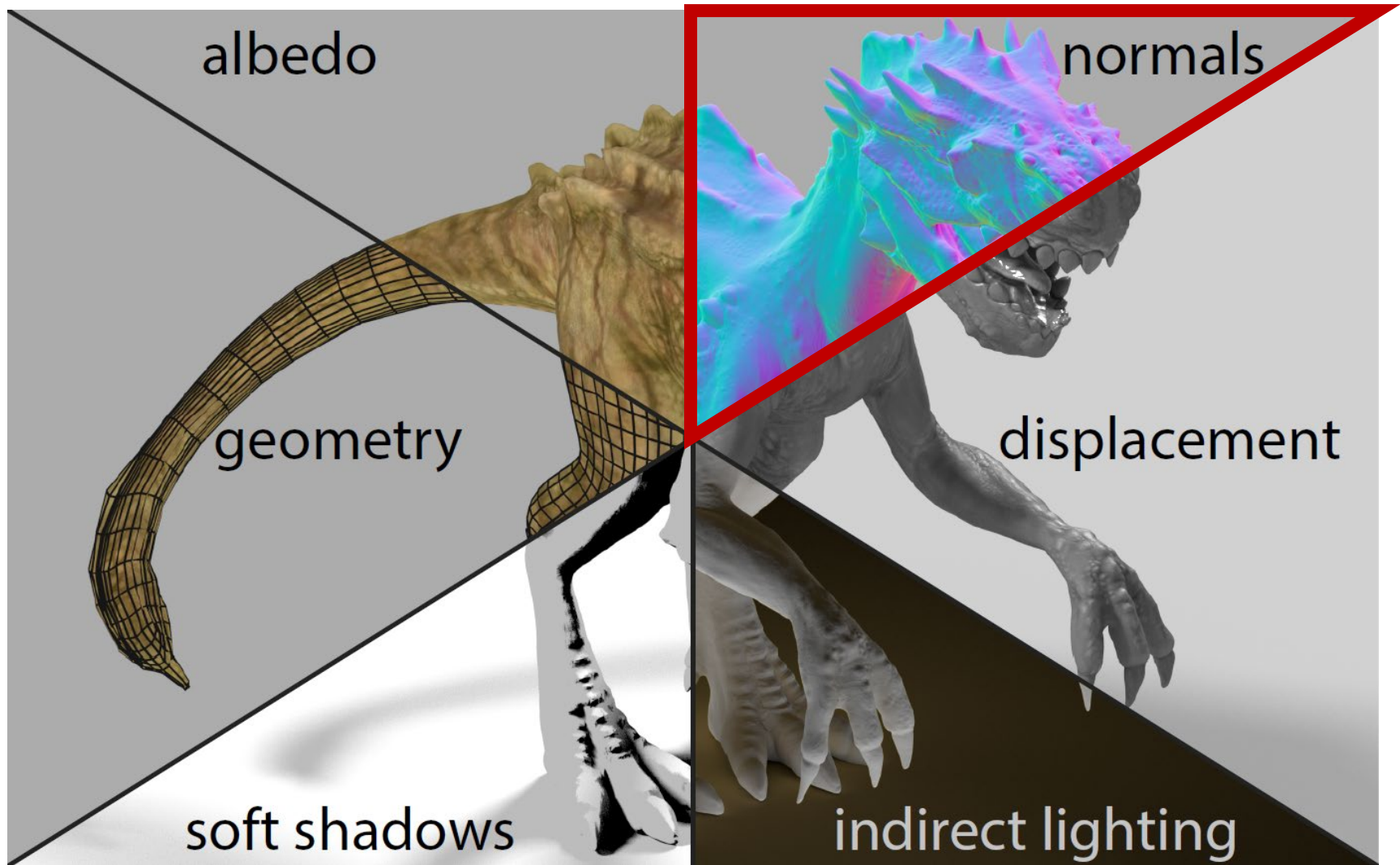




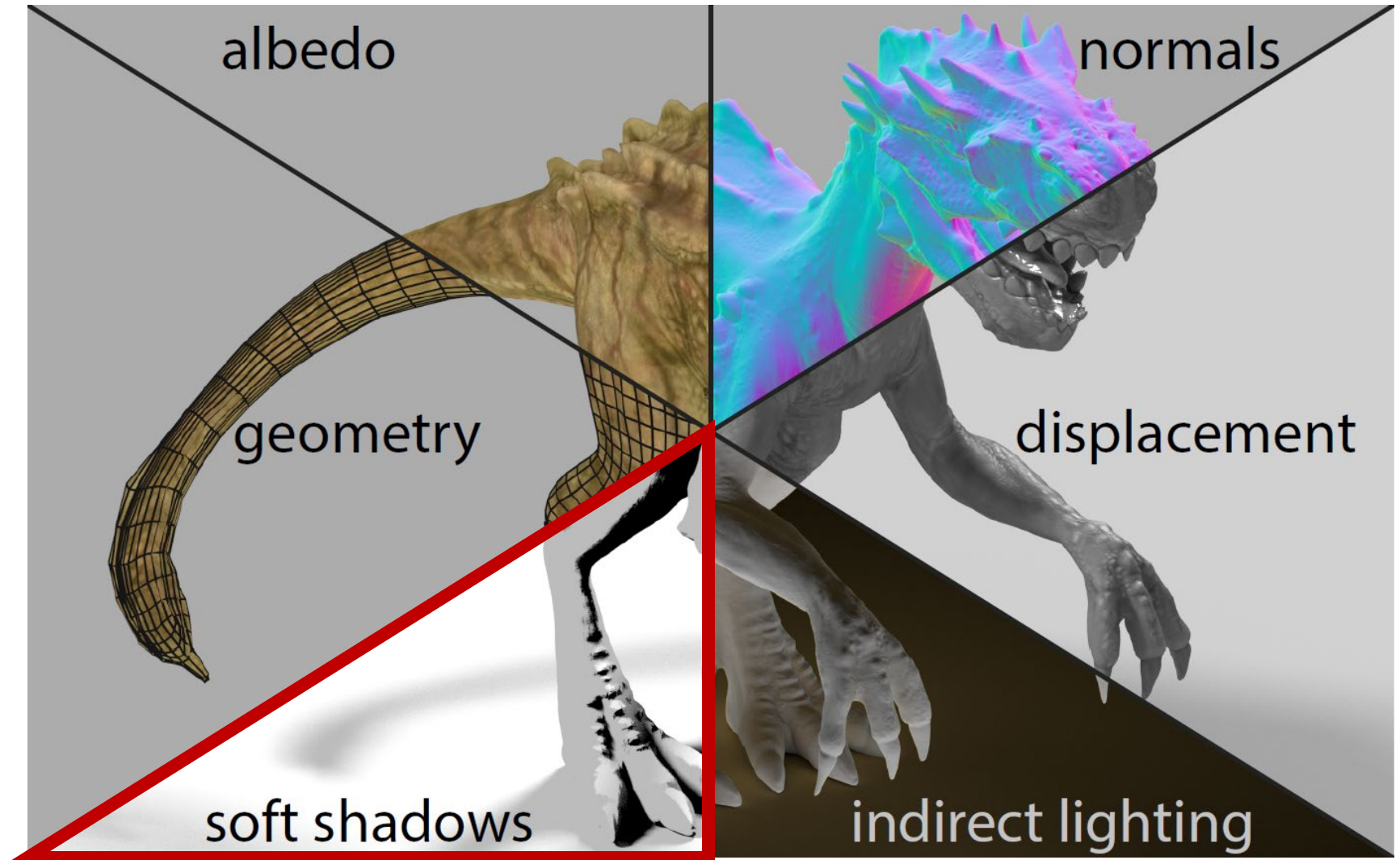
# Texture Mapping - Introduction



# Texture Mapping - Introduction

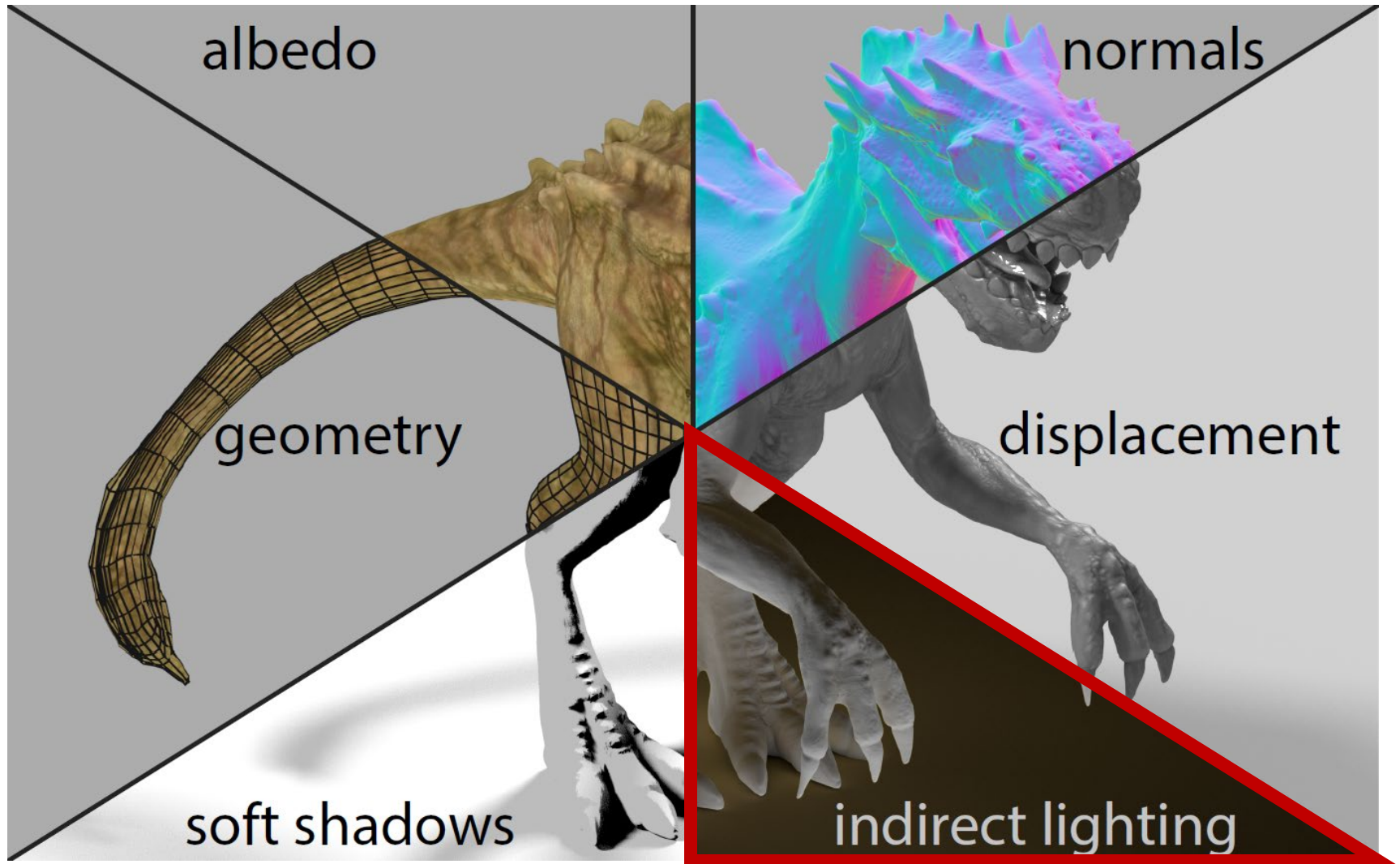


# Texture Mapping - Introduction

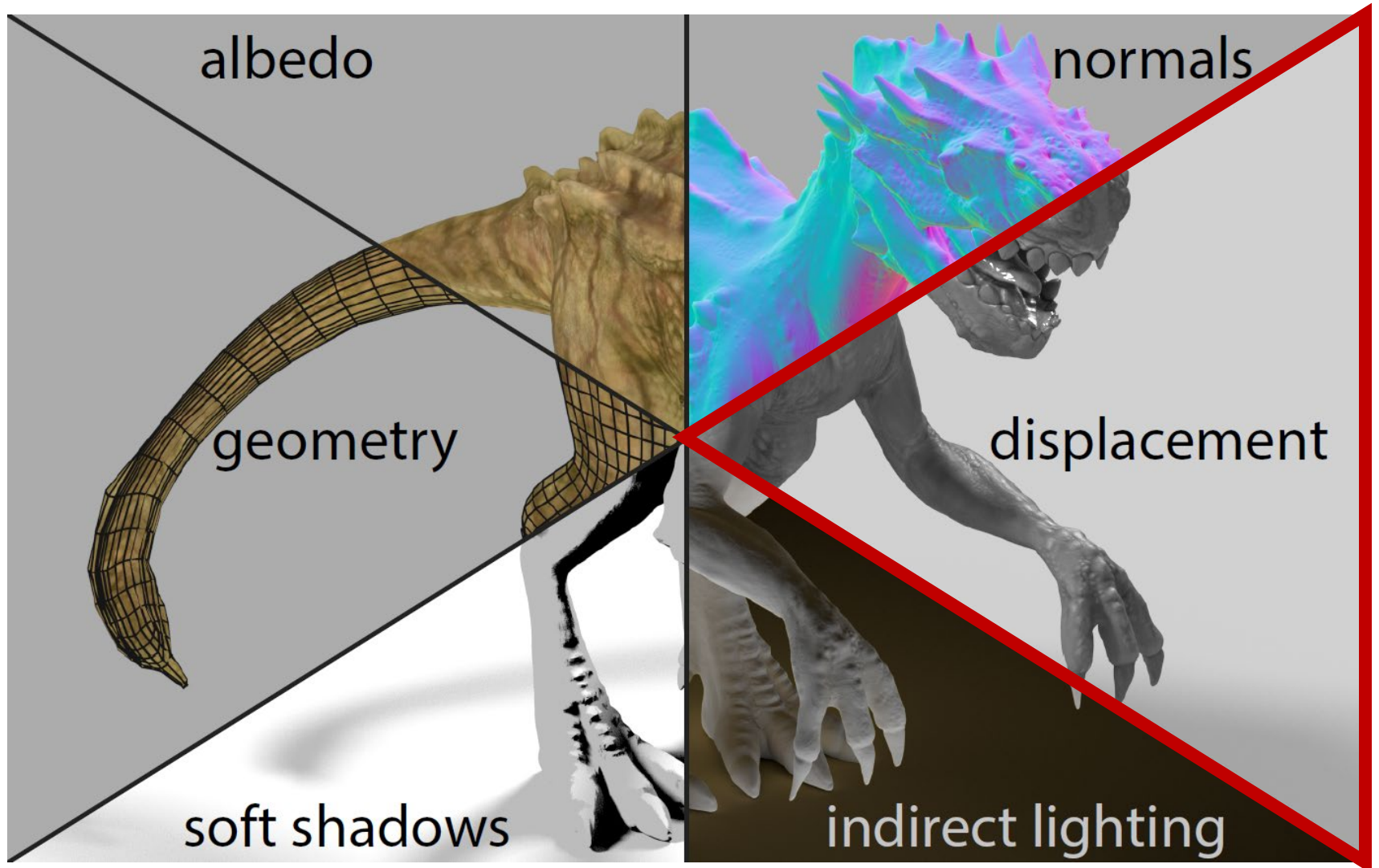




# Texture Mapping - Introduction



# Texture Mapping - Introduction



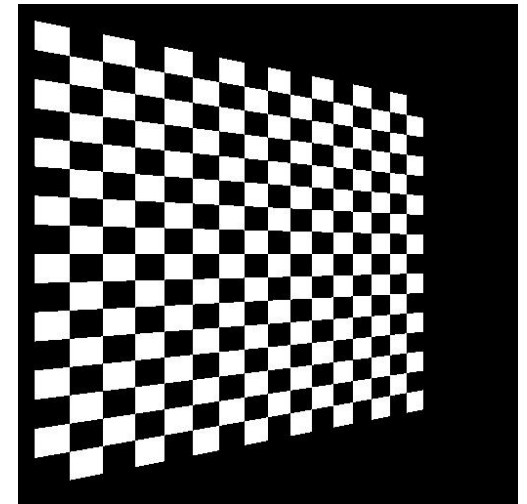
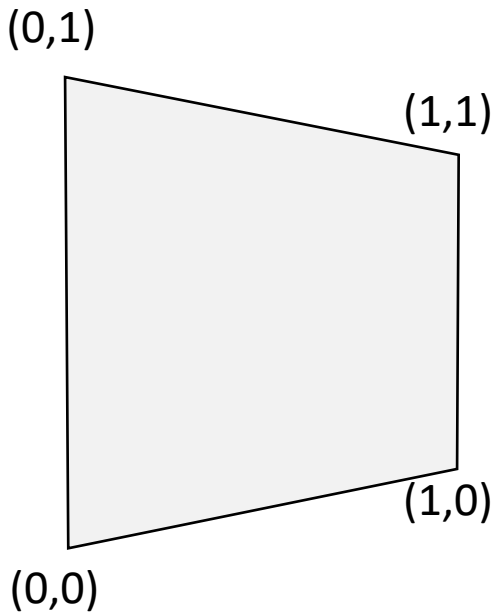
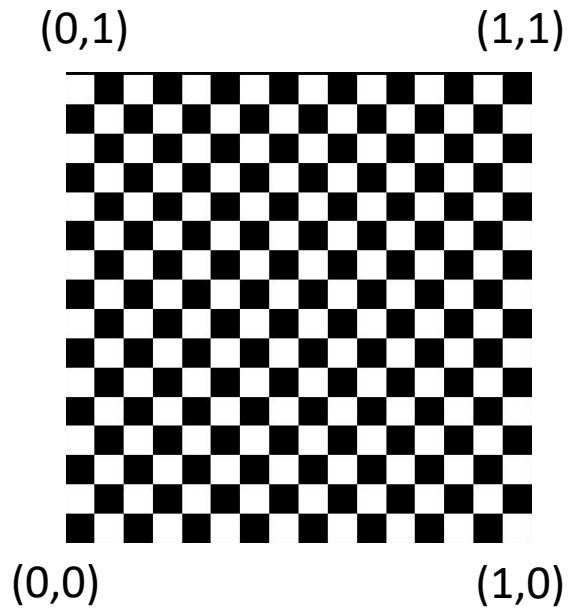


# Texture Mapping

- Mapping in 2D:
  - Texture image of size  $(n_x, n_y)$
  - Constraints on some architectures (powers of 2)
  - Texture coordinates “s” and “t” for accessing texture images
    - $(s, t, r)$  in 3D and
    - $(s, t, r, q)$  homogeneous texture coordinates
  - Assign to every geometric point  $(x, y, z)$  on the polygon **P** a texture coordinate  $(s, t)$ :  
  
→  $F: P \in \mathbb{R}^3 \rightarrow [0,1]^2 \in \mathbb{R}^2$
- Simple procedure:
  1. for every vertex assign  $(s, t)$ .
  2. For interior points assign  $(s, t)$  by interpolation.

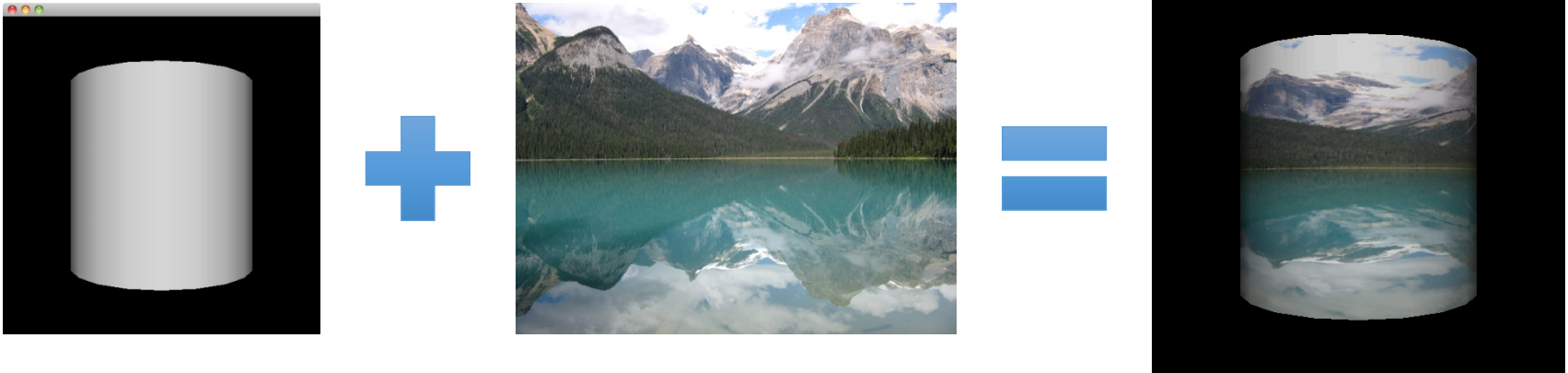
# Texture Mapping

• Texture + Quad = Image

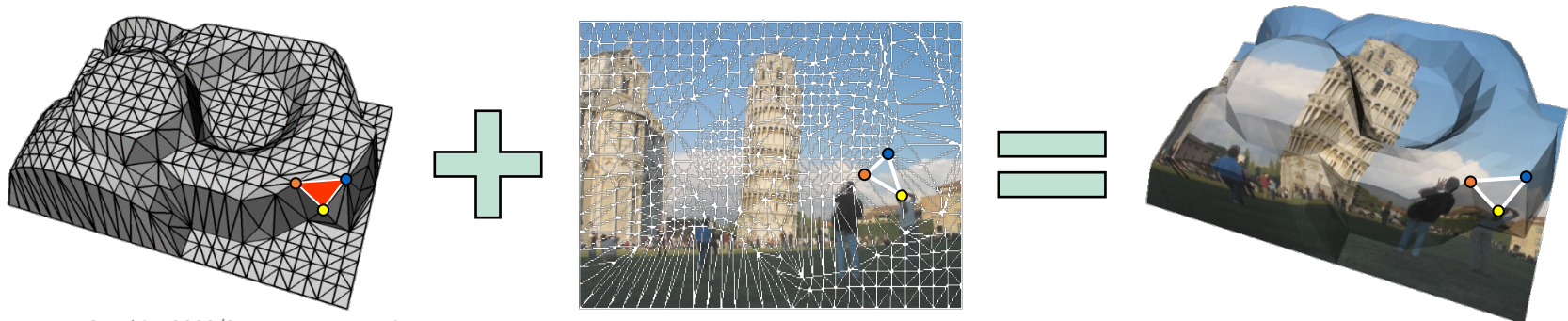


# Parameterization

- Texture coordinates → **Parameterization**
- Simple parameterization

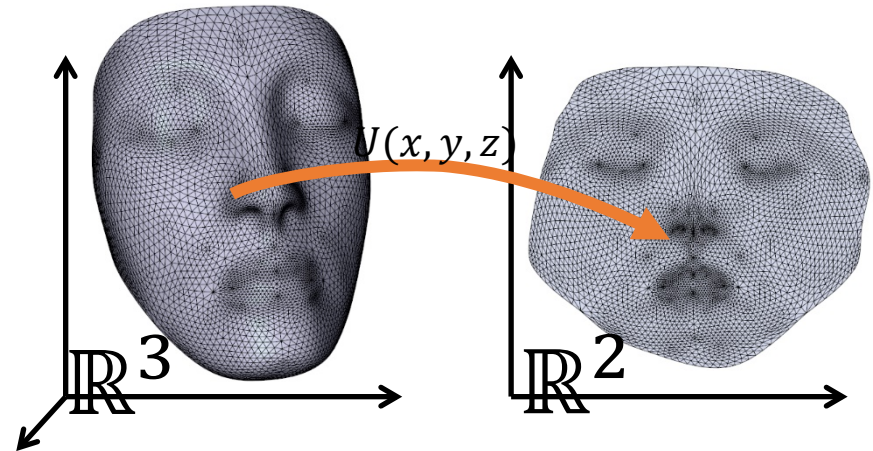


- difficult parameterization



# Parameterization

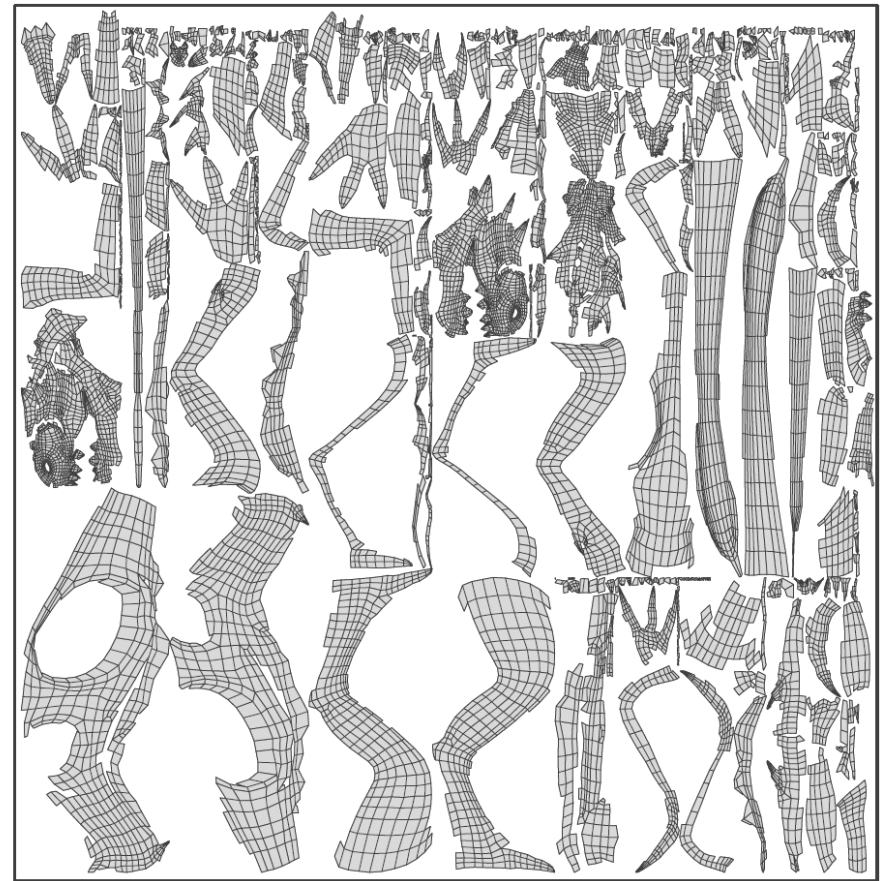
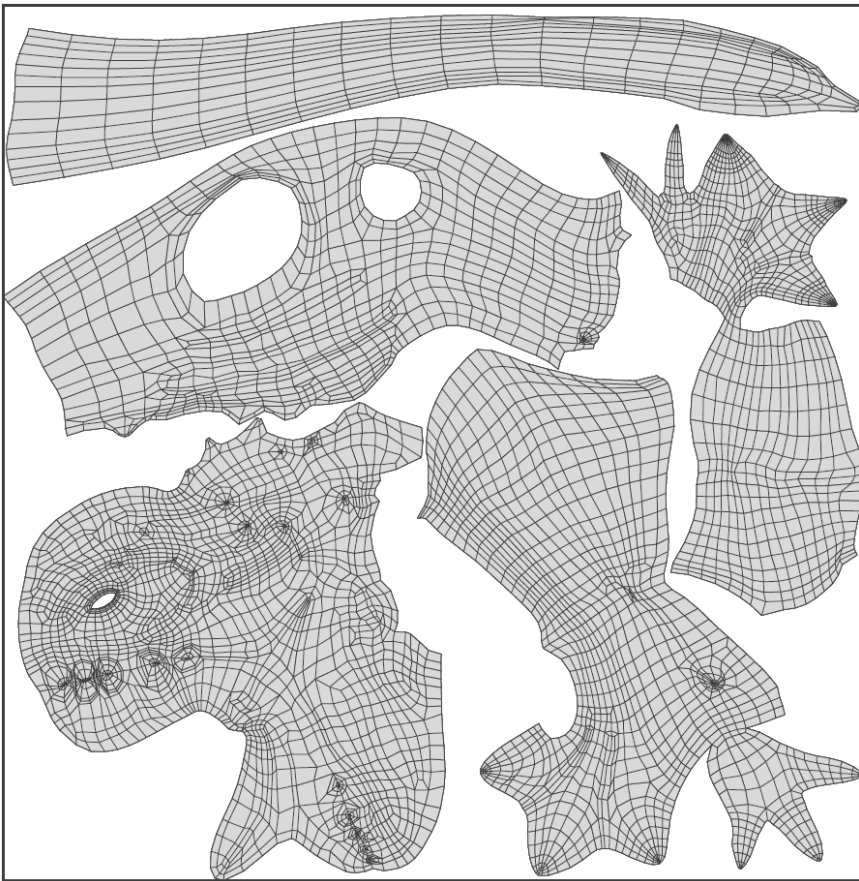
- Find a mapping from 3D surface to 2D plane (or vice versa)
- Long standing problem
- solutions available in modeling programs, often not robust
- → lecture „**Geometry Processing**“





# Parameterization

- **Texture Atlas:**  
not one single texture, but fragmented textures for object parts

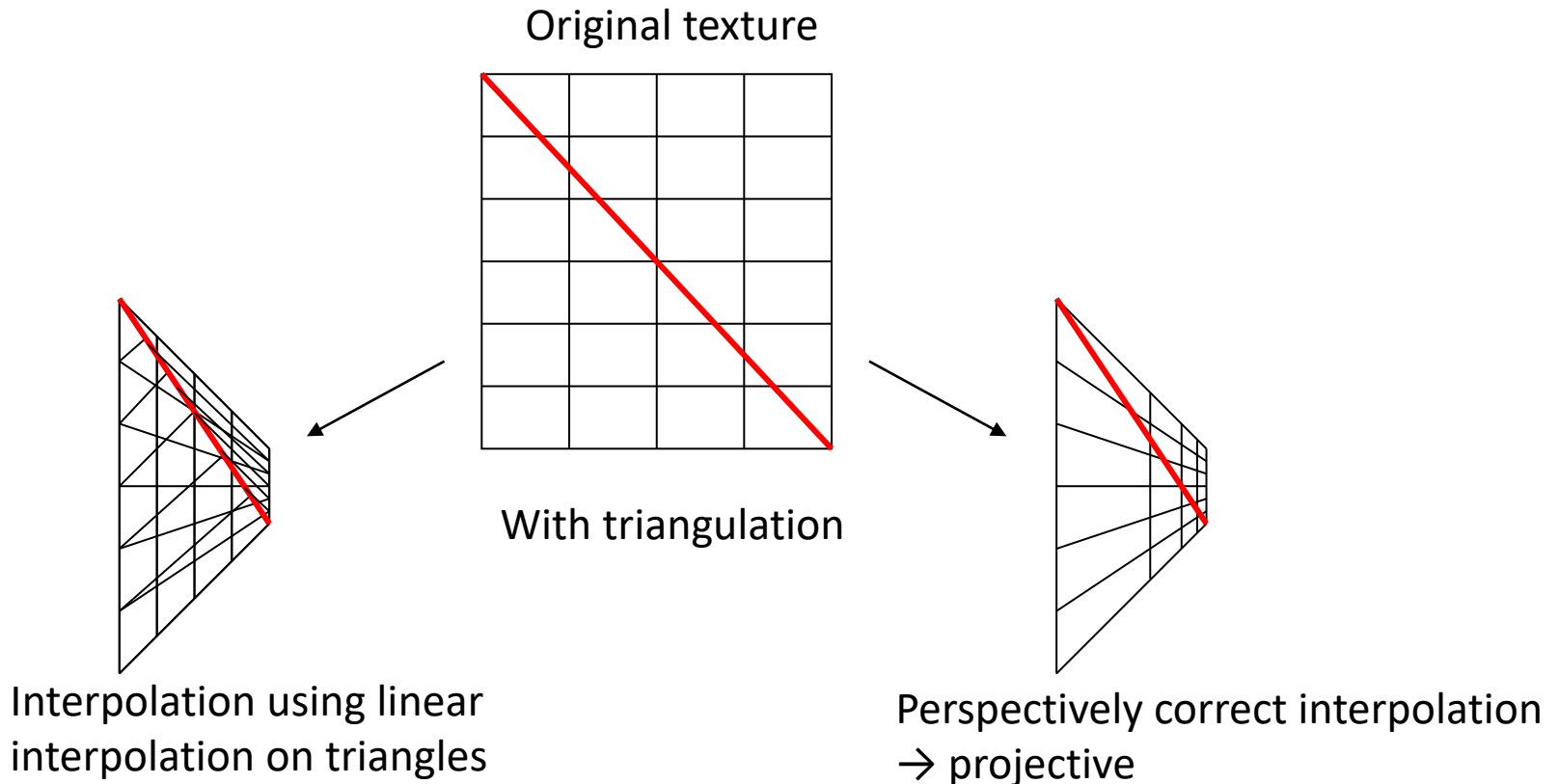




# Texture Mapping for Rasterized Triangles

- Interpolation Problem

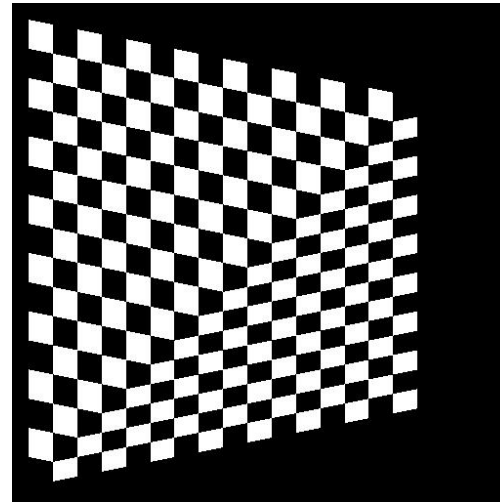
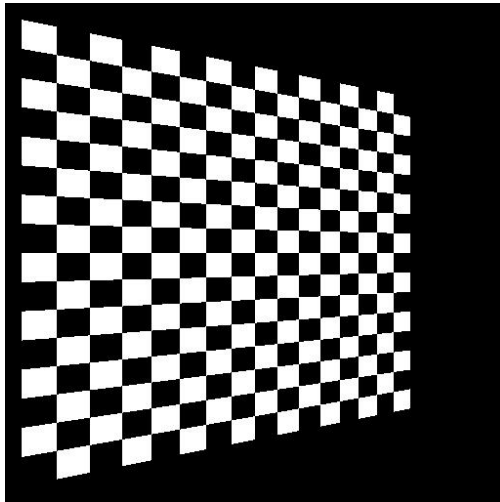
- Standard interpolation method at rasterization stage (linear interpolation) results in distorted images!
- Reason: Does not consider the distortion of the perspective transformation!



# Texture Mapping for Rasterized Triangles

- Correct

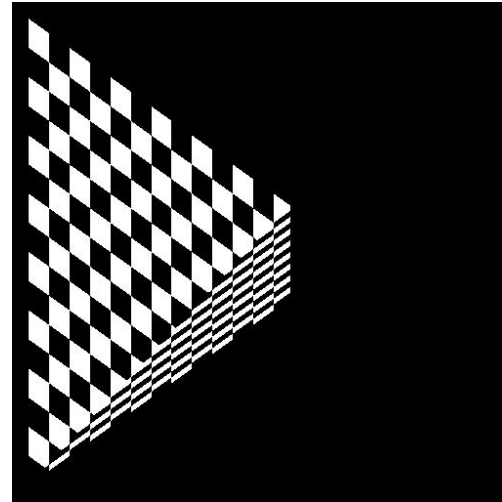
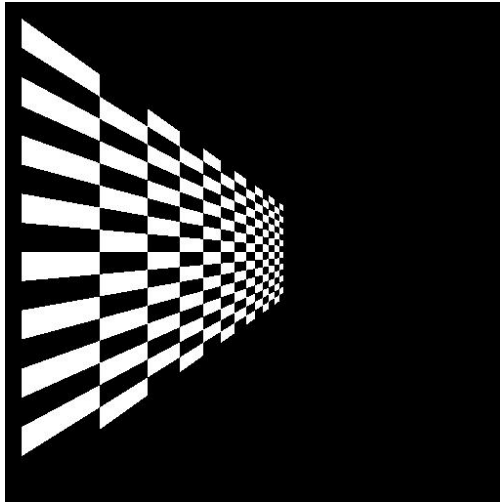
wrong



# Texture Mapping for Rasterized Triangles

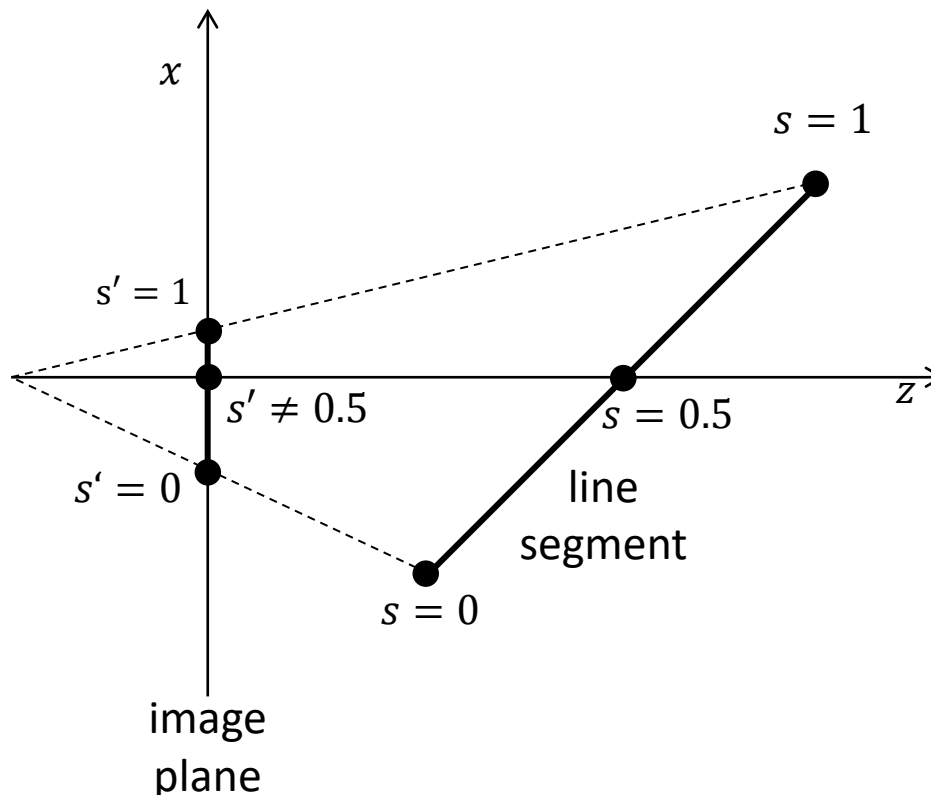
- Correct

very wrong



# Texture Mapping for Rasterized Triangles

- Perspective interpolation – problem statement
  - Example: line segment not parallel to image plane:
  - $s$ : texture coordinate in world space,  $s'$ : texture coordinate in screen space
  - Linear interpolation of  $s'$  in screen space does not match interpolation of  $s$  in world coordinates.



# Texture Mapping for Rasterized Triangles

- Perspective Interpolation

- Needed: Mapping  $s' \rightarrow s$  that implements perspective correct linear interpolation in screen space
- Solution: consider the division by  $z$ !
- following derivation from [http://www.comp.nus.edu.sg/~lowkl/publications/lowk\\_persp\\_interp\\_techrep.pdf](http://www.comp.nus.edu.sg/~lowkl/publications/lowk_persp_interp_techrep.pdf)



# Texture Mapping for Rasterized Triangles

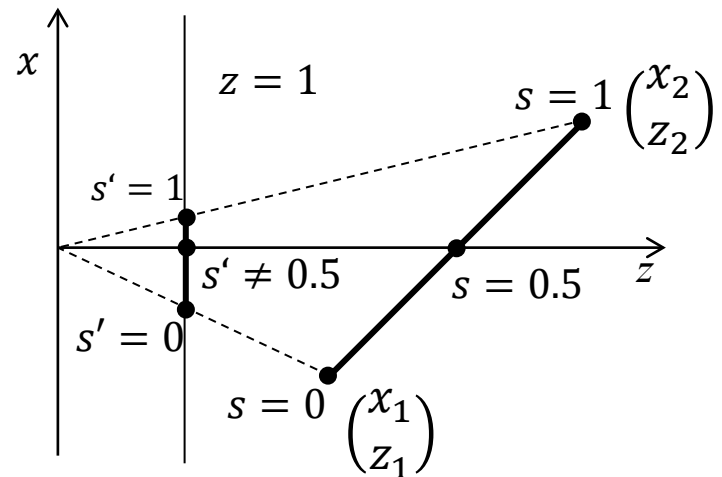
- $s$ : relative position in world space,  $s'$  in image space
- In world space, we describe the line segment as:

$$\begin{pmatrix} x \\ z \end{pmatrix} = \begin{pmatrix} x_1 \\ z_1 \end{pmatrix} + s \begin{pmatrix} x_2 - x_1 \\ z_2 - z_1 \end{pmatrix}$$

- in image space:

$$x' = \frac{x_1}{z_1} + s' \left( \frac{x_2}{z_2} - \frac{x_1}{z_1} \right)$$

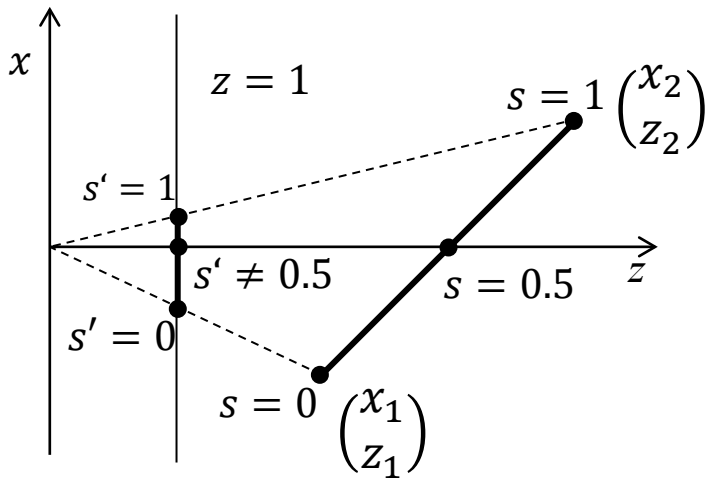
- Obviously  $s'$  is not the same as  $s$ !



# Texture Mapping for Rasterized Triangles

- During rasterization, we know  $s'$ , and need to derive  $s$  from  $s'$
- with some arithmetics, we find

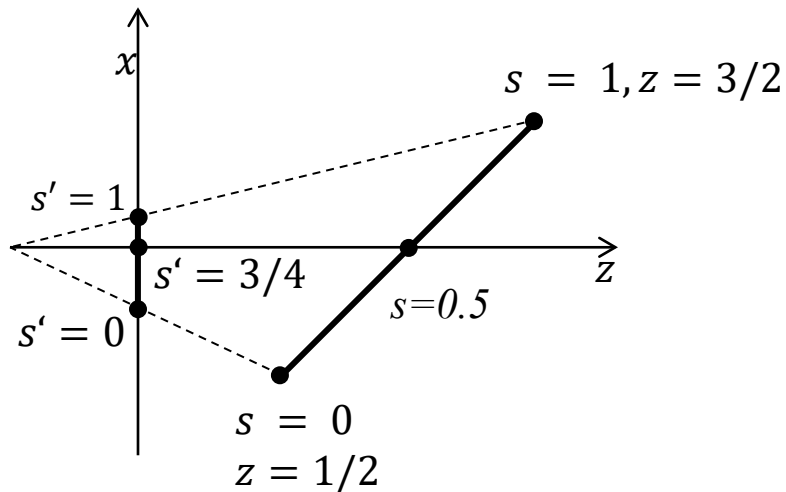
$$s = \frac{s'z_1}{s'z_1 + (1 - s')z_2}$$



# Texture Mapping for Rasterized Triangles

- Example

$$\bullet \ S' = \frac{3}{4} \rightarrow S = \frac{\frac{3}{4}Z_1}{\frac{3}{4}Z_1 + \frac{1}{4}Z_2} = \frac{1}{2}$$



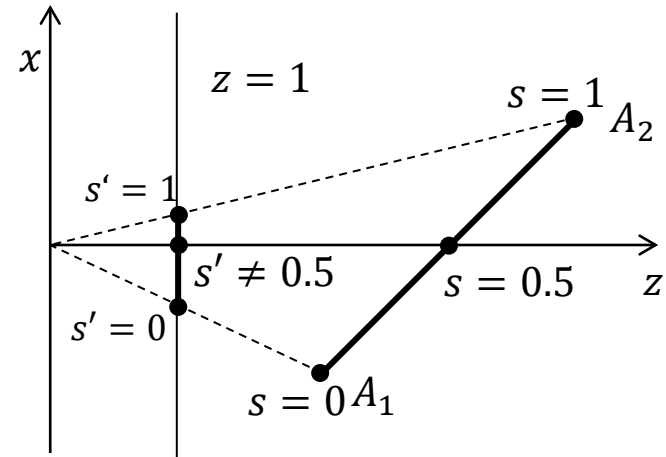
# Texture Mapping for Rasterized Triangles

- for arbitrary attributes  $A$  along a line:

- $z$ -values  $z_1$  and  $z_2$
- attribute values  $A_1$  and  $A_2$

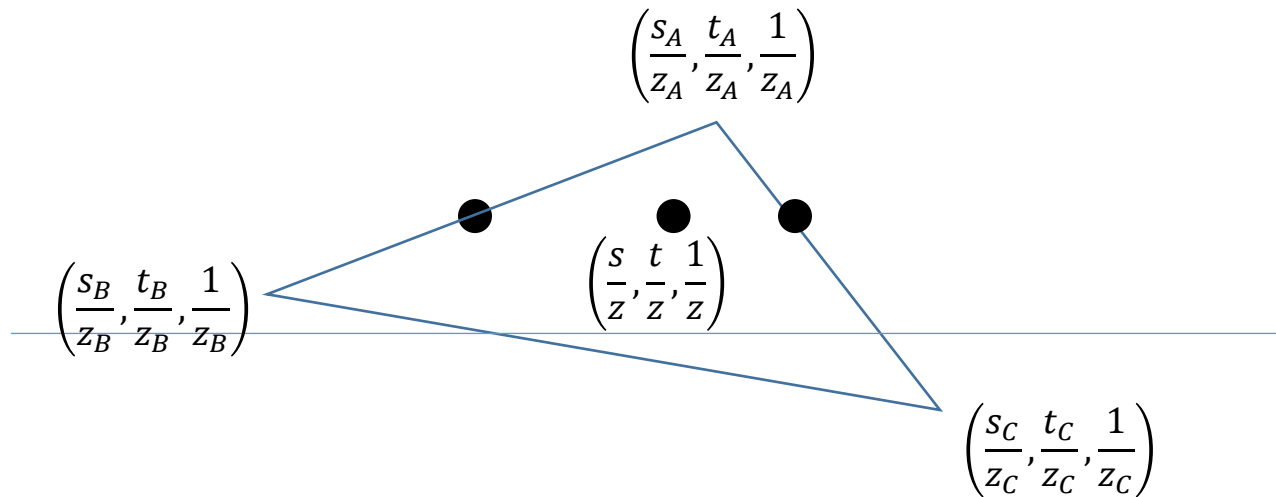
- $$A(s) = A_1 + s(A_2 - A_1) = \dots = \frac{\frac{A_1}{z_1} + s'(\frac{A_2}{z_2} - \frac{A_1}{z_1})}{\frac{1}{z_1} + s'(\frac{1}{z_2} - \frac{1}{z_1})}$$

- Interpolate  $A/z$  and  $1/z$
- divide to get interpolated  $A$



# Texture Mapping for Rasterized Triangles

- From this, we can derive an approach for interpolating texture coordinates
  - interpolate  $s/z$ ,  $t/z$ , and  $1/z$  during rasterization
  - Per pixel:  $(s/z)/(1/z), (t/z)/(1/z) \rightarrow (s, t)$



- Also works for arbitrary attributes

# Texture Mapping

- In OpenGL / WebGL:
  - 1D, 2D and 3D textures
  - textures can have luminance only (grey value), luminance plus alpha, color, or color plus alpha
  - 8bit per channel, 16bit per channel, or float values
  - are sampled in a shader using a **sampler** object
  - homogeneous texture coordinates  $(s, t, r, q)$
  - newer OpenGL also supports compressed textures

# Texture Mapping

- In WebGL


```
if (!texHandle) {  
    var image = document.getElementById(„mytexture“);  
    texHandle = gl.createTexture();  
    gl.bindTexture(gl.TEXTURE_2D, texHandle);  
    gl.texImage2D(gl.TEXTURE_2D, 0, gl.RGBA, gl.RGBA,  
                  gl.UNSIGNED_BYTE, image);  
} else  
    gl.bindTexture(gl.TEXTURE_2D, texHandle);
```

- pixel shader

```
...  
uniform sampler2D texture;  
varying vec2 uv; // texture coordinate  
  
void main(void) {  
    ...  
    // finally, apply texture by multiplication  
    gl_FragColor *= texture2D(texture, uv);  
}
```

# Texture Mapping Demo

Textures



Texture Magnification

☒ NEAREST

☐ LINEAR

Texture Minification

☒ NEAREST - No MIPMap

☐ LINEAR - No MIPMap

☐ NEAREST\_MIPMAP\_NEAREST

☐ LINEAR\_MIPMAP\_NEAREST

☐ NEAREST\_MIPMAP\_LINEAR

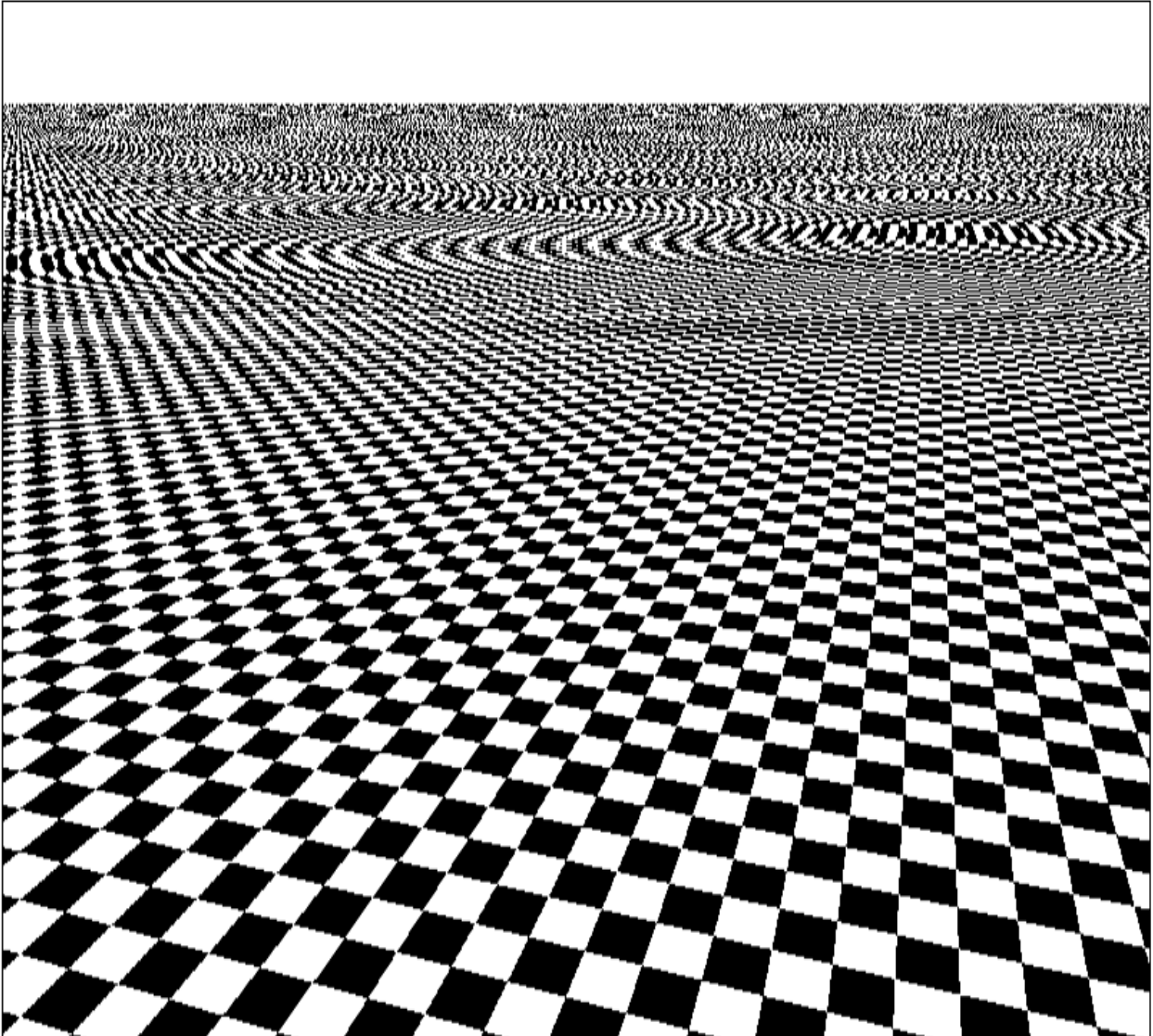
☐ LINEAR\_MIPMAP\_LINEAR

Patch

☒ Infinite Patch

☐ Mirror Patch

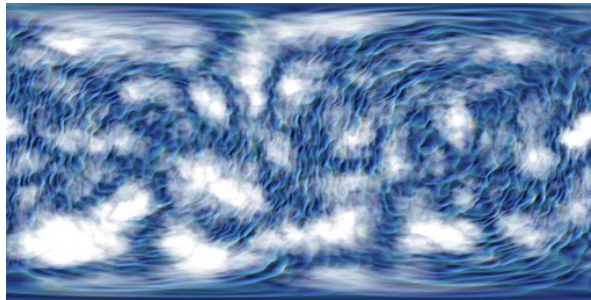
☐ Normal Map



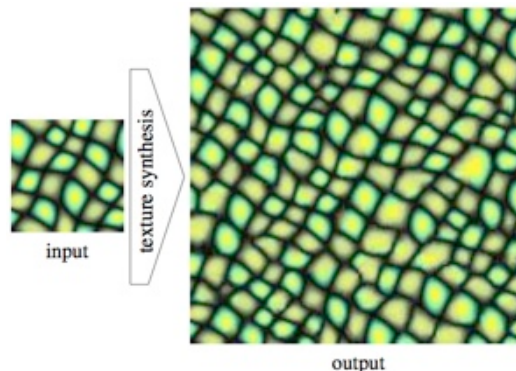


# Procedural Texture Generation

- Textures can come from an image file, e.g. jpg
- or can be generated by a procedure
  - on the fly in a shader
    - often based on fractal noise or turbulence functions (see later)



- → Texture synthesis: generate arbitrarily large high-quality texture from a small input sample.



# Procedural Texture Generation

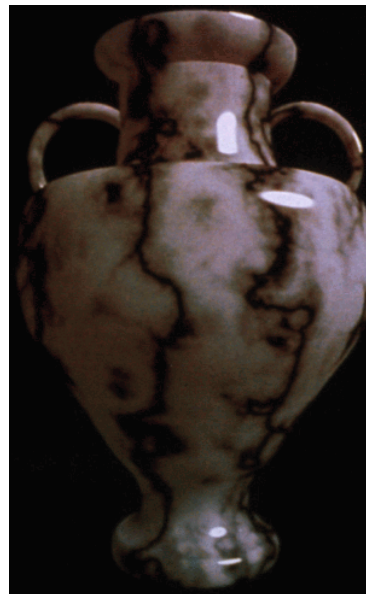
- regular stripes dark/bright brown
- Stripe width/distance: varies over years
- Shape of stripes slightly unregular
- Less or more irregular patterns possible
- Example:  
<https://www.shadertoy.com/view/ldscDM>



freestocktextures.com

# Procedural Texture Generation

- Procedural texture generation
  - Computer generated texture image (1D, 2D or 3D) created using an algorithm.
  - Natural appearance requires some randomness, but also structure
  - All based on **Noise Functions**



```

/* Copyrighted Pixar 1988 */
/* From the RenderMan Companion p. 355 */
/* Listing 16.19 Blue marble surface shader */

/*
 * blue_marble(): a marble stone texture in shades of blue
 * surface
 */

blue_marble(
    float Ks = .4,
           Kd = .6,
           Ka = .1,
           roughness = .1,
           txtscale = 1,
           color specularcolor = 1)
{
    point PP; /* scaled point in shader space */
    float csp; /* color spline parameter */
    point Nf; /* forward-facing normal */
    point V; /* for specular() */
    float pixsize, twice, scale, weight, turbulence;

    /* Obtain a forward-facing normal for lighting calculations. */
    Nf = faceforward( normalize(N), 1);
    V = normalize(-1);

    /*
     * Compute "turbulence" a la [PERLINS]. Turbulence is a sum of
     * "noise" components with a "fractal" 1/f power spectrum. It gives the
     * visual impression of turbulent fluid flow (for example, as in the
     * formation of blue_marble from molten color splines). Use the
     * surface element area in texture space to control the number of
     * noise components so that the frequency content is appropriate
     * to the scale. This prevents aliasing of the texture.
     */
    PP = transform("shader", P) * txtscale;
    pixsize = sqrt(area(PP));
    twice = 2 * pixsize;
    turbulence = 0;
    for (scale = 1; scale > twice; scale *= 2)
        turbulence += scale * noise(PP/scale);

    /* Gradual fade out of highest-frequency component near limit */
    if (scale > pixsize) {
        weight = (scale / pixsize) - 1;
        weight = clamp(weight, 0, 1);
        turbulence += weight * scale * noise(PP/scale);
    }

    /*
     * Magnify the upper part of the turbulence range 0.75:1
     * to fill the range 0:1 and use it as the parameter of
     * a color spline through various shades of blue.
     */
    csp = clamp(4 * turbulence - 3, 0, 1);
    Ci = color spline(csp,
        color (0.25, 0.25, 0.35), /* pale blue */
        color (0.25, 0.25, 0.35), /* pale blue */
        color (0.20, 0.20, 0.30), /* medium blue */
        color (0.20, 0.20, 0.30), /* medium blue */
        color (0.20, 0.20, 0.30), /* medium blue */
        color (0.25, 0.25, 0.35), /* pale blue */
        color (0.25, 0.25, 0.35), /* pale blue */
        color (0.15, 0.15, 0.25), /* medium dark blue */
        color (0.15, 0.15, 0.25), /* medium dark blue */
        color (0.10, 0.10, 0.20), /* dark blue */
        color (0.10, 0.10, 0.20), /* dark blue */
        color (0.25, 0.25, 0.35), /* pale blue */
        color (0.10, 0.10, 0.20) /* dark blue */
    );

    /* Multiply this color by the diffusely reflected light. */
    Ci *= Ka*ambient() + Kd*diffuse(Nf);

    /* Adjust for opacity. */
    Oi = Os;
    Ci = Ci * Oi;

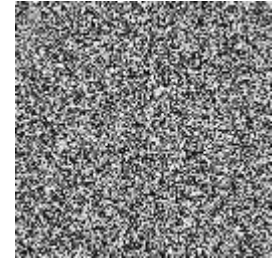
    /* Add in specular highlights. */
    Ci += specularcolor * Ks * specular(Nf,V,roughness);
}

```

# Procedural Texture Generation

- **Noise Functions:**

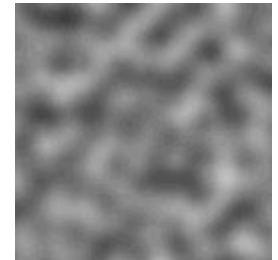
- “White noise”: Assign random color for every point
  - no coherence
  - not helpful for procedural textures
  - **coherency** required



Non coherent

- **Coherent Noise**

- Method for generating coherent noise over space.
- Coherent means: the function values change smoothly.



Coherent

- **First Approach**

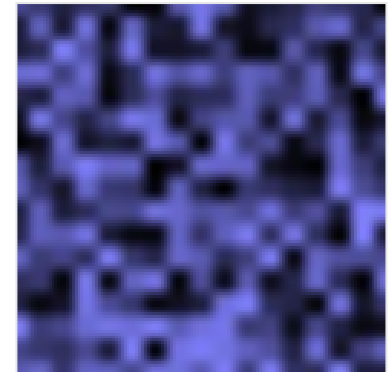
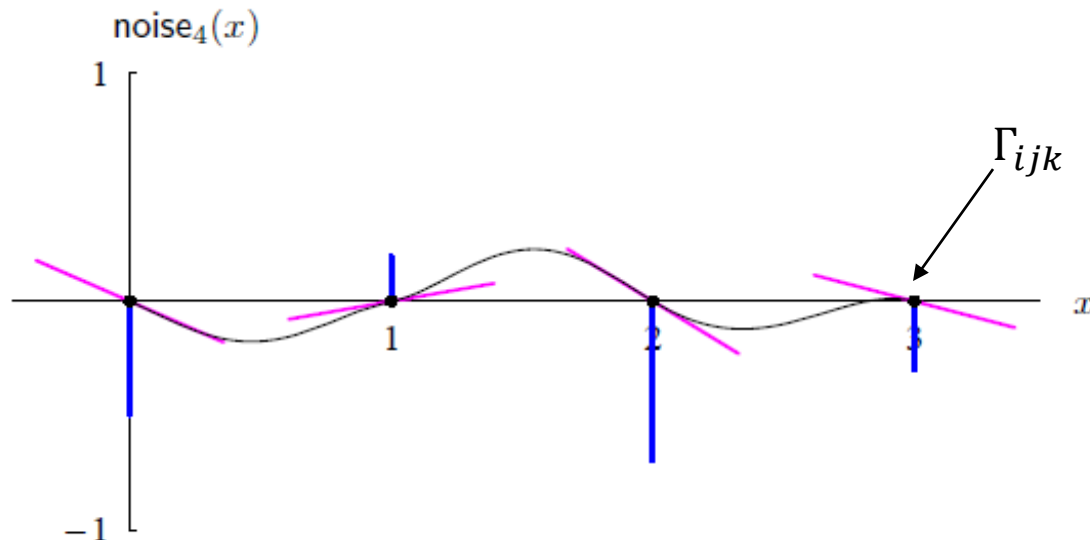
- choose random values on a grid
- interpolate
- grid size corresponds to **noise frequency**

Images by Matt Zucker

# Procedural Texture Generation

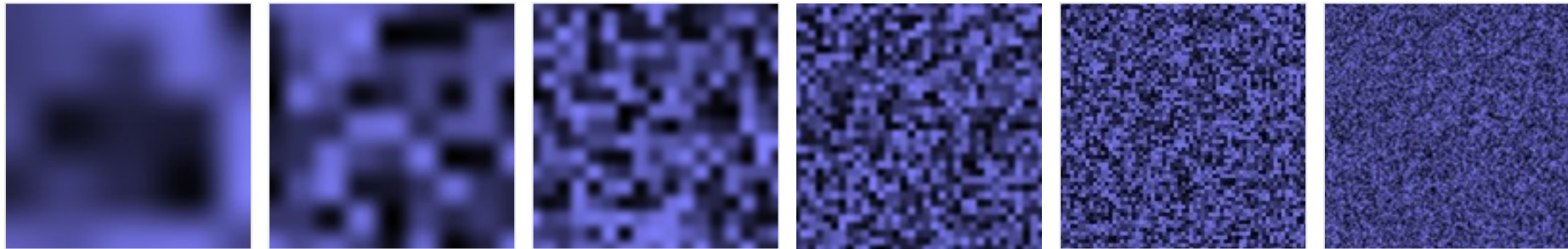
- Better approach, proved very practical: **Perlin Noise**
  - At grid points  $(i, j, k)$  choose random **gradient**  $\Gamma_{ijk}$ , set values to zero
  - $\Gamma_{ijk}$  is determined from  $(i, j, k)$  using an array of precomputed random gradient values  $G[]$  and a **hash function**  $\phi()$  as:
$$\Gamma_{ijk} = G\left(\phi\left(i + \phi(j + \phi(k))\right)\right)$$

→ „pseudorandom“ gradient values, very fast to compute
  - Then, these grid point gradients are interpolated

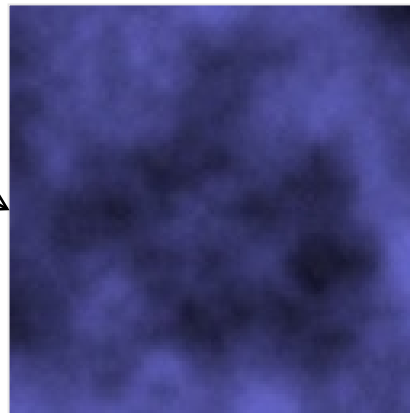
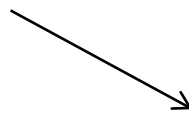


# Procedural Texture Generation

- Simple Perlin Noise is boring
- Gets interesting by adding noise of varying frequency:



Sum of all layers

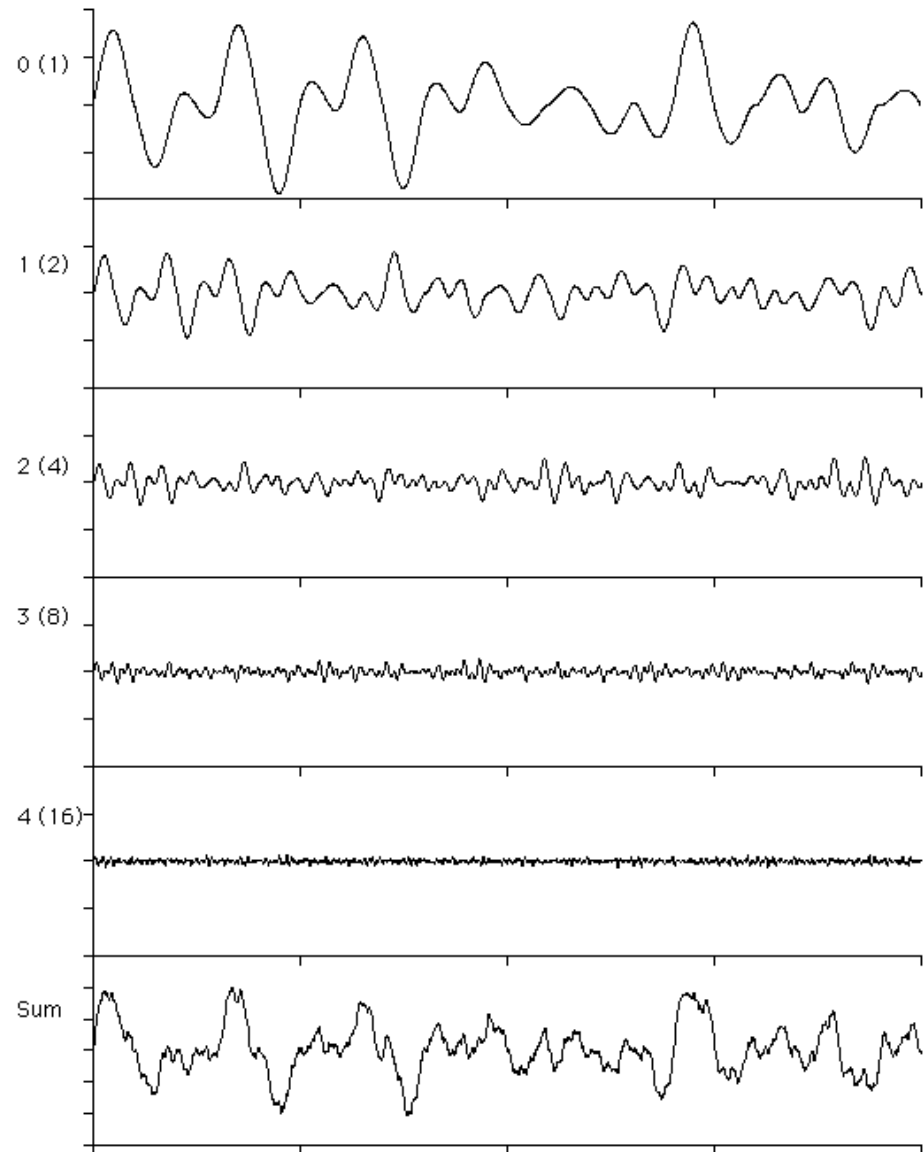


# Procedural Texture Generation

- Turbulence

- Many natural textures contain repeating features of different sizes
- Perlin pseudo fractal “turbulence” function
- Effectively adds scaled copies of noise function on top of itself

$$n_t(x) = \sum_i \frac{|n(2^i x)|}{2^i}$$



# Procedural Texture Generation

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 */
blue_marble(
```

```
PP = transform("shader", P) * txtscale;
pixelsize = sqrt(area(PP));
twice = 2 * pixelsize;
turbulence = 0;
for (scale = 1; scale > twice; scale /= 2)
    turbulence += scale * noise(PP/scale);
```

```
/* Gradual fade out of highest-frequency component near limit */
if (scale > pixelsize) {
    weight = (scale / pixelsize) - 1;
    weight = clamp(weight, 0, 1);
    turbulence += weight * scale * noise(PP/scale);
}
```

```
color (0.15, 0.15, 0.25) /* medium dark blue */
color (0.15, 0.15, 0.25) /* medium dark blue */
color (0.10, 0.10, 0.20) /* dark blue */
color (0.10, 0.10, 0.20) /* dark blue */
color (0.25, 0.25, 0.35) /* pale blue */
color (0.10, 0.10, 0.20) /* dark blue */
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/* Multiply this color by the diffusely reflected light. */
Ci *= Ka*ambient() + Kd*diffuse(Nf);

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Oi = Os;
Ci = Ci * Oi;

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```



# Procedural Texture Generation

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    weight = (scale / pixelsize) - 1;
    weight = clamp(weight, 0, 1);
    turbulence += weight * scale * noise(PP/scale);
}
```

```
color (0.15, 0.15, 0.25), /* medium dark blue */
color (0.15, 0.15, 0.25), /* medium dark blue */
color (0.10, 0.10, 0.20), /* dark blue */
color (0.10, 0.10, 0.20), /* dark blue */
color (0.25, 0.25, 0.35), /* pale blue */
color (0.10, 0.10, 0.20) /* dark blue */
);

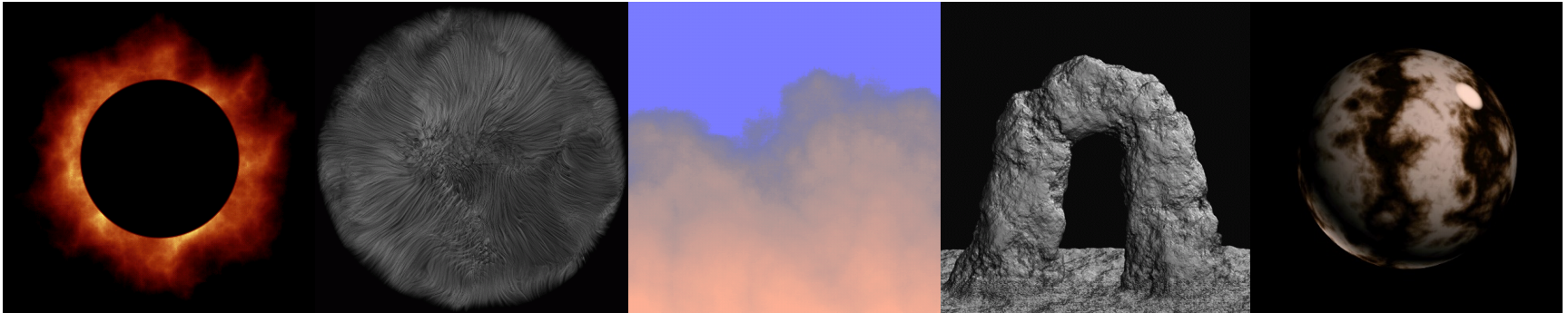
/* Multiply this color by the diffusely reflected light. */
Ci *= Ka*ambient() + Kd*diffuse(Nf);

/* Adjust for opacity. */
Oi = Oi;
Ci = Ci * Oi;

/* Add in specular highlights. */
Ci += specularcolor * Ks * specular(Nf,V,roughness);
}
```

# Procedural Texture Generation

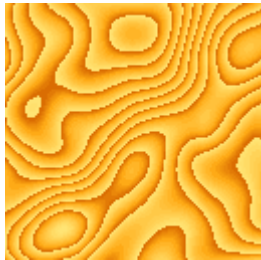
- Perlin noise
  - Solid texture
  - Based on gradient noise
    - Generate an n-dimensional lattice of random gradients
    - The noise value is interpolated in the lattice cells, e.g. using linear or cosine interpolation.
  - Gradient noise is conceptually different than value or wavelet noise.



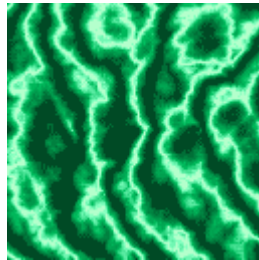
<http://www.noisemachine.com/talk1/>

# Texture Functions: Perlin noise

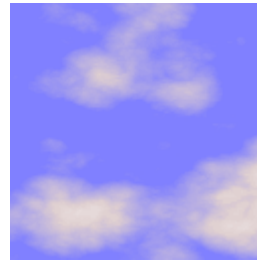
- other examples:



wood

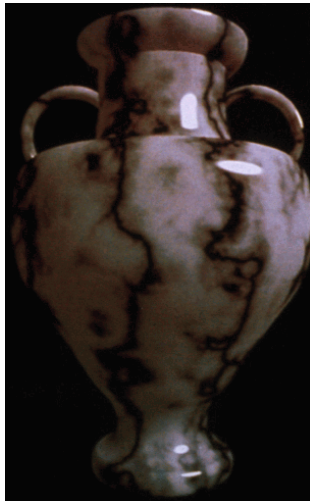


marble



clouds

Images by Matt Zucker



see also ShaderToy „Perlin Noise“, e.g.

<https://www.shadertoy.com/view/Md3SzB>

<https://www.shadertoy.com/view/4tdSWr>

Image by Ken Perlin

# Texture Functions

- wood shader



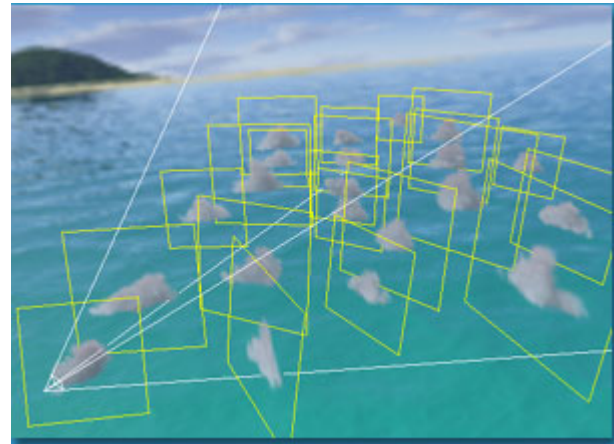
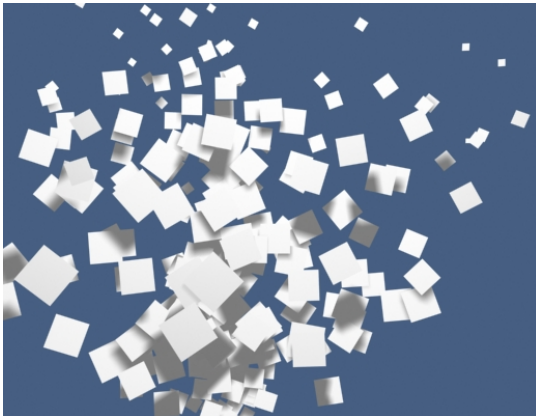
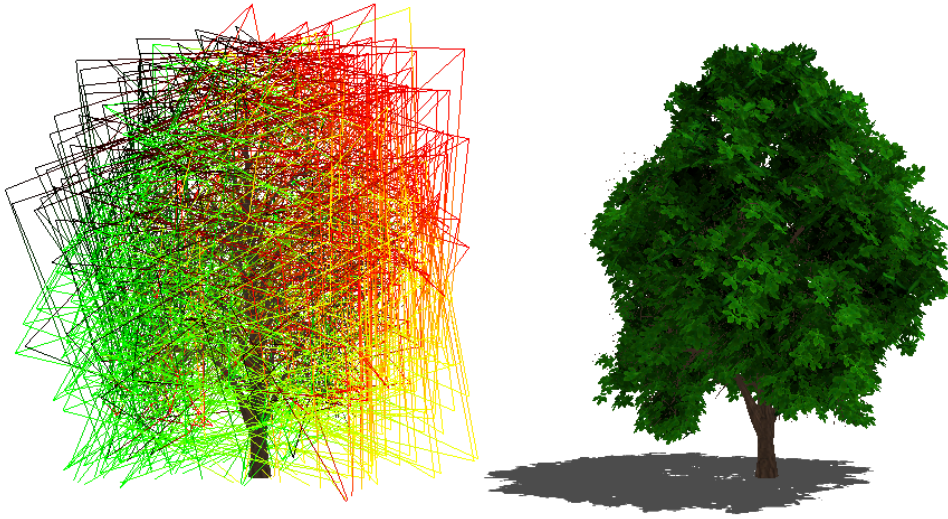
<http://flycooler.com/>



[bertramguitars.com](http://bertramguitars.com)

# Texture Mapping

- many other applications for textures



# Textures Beyond Wallpaper: Normal Maps

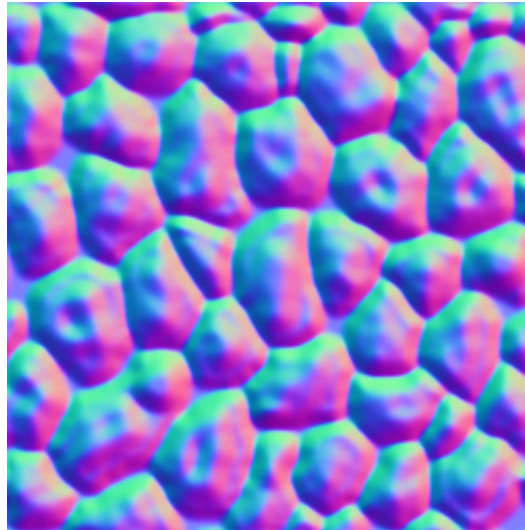
- Non-Graphics Application: „Solarscreens“





# Textures Beyond Wallpaper: Normal Maps

- Normal Map
  - texture with 3D normals encoded in RGB
  - 8 Bit per component sufficient
    - but also 3x10 Bit, 4x16 Bit unsigned, floating point
  - $[-1,1]$  to  $[0;1]$ 
    - $R = x/2 + 0.5$ ,  $G = y/2 + 0.5$ , ...
    - $x = 2R-1$ , ...

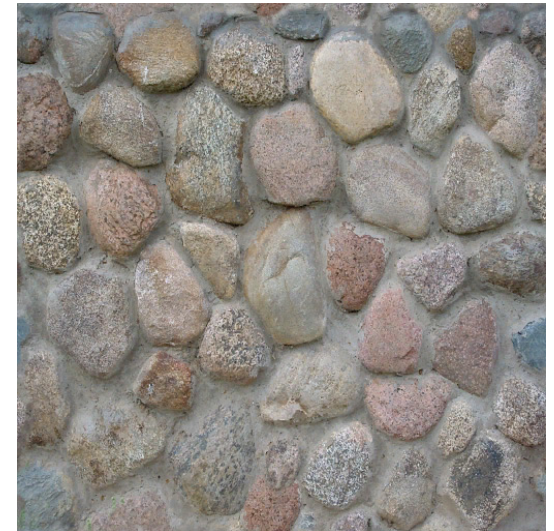
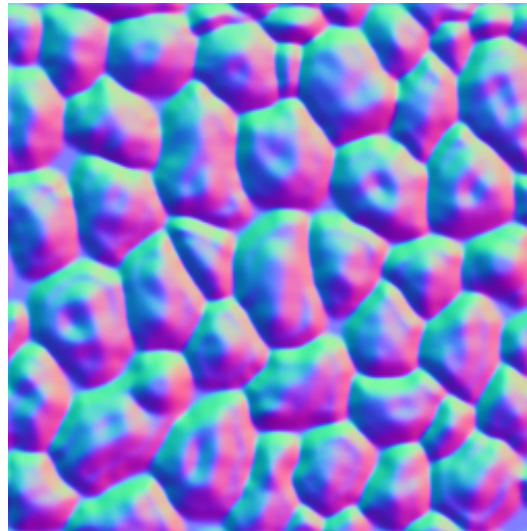
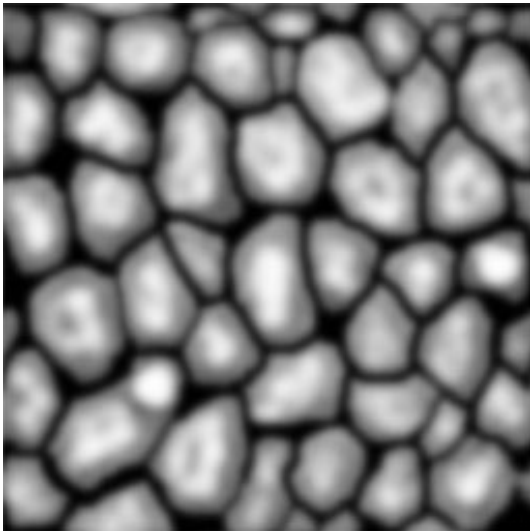
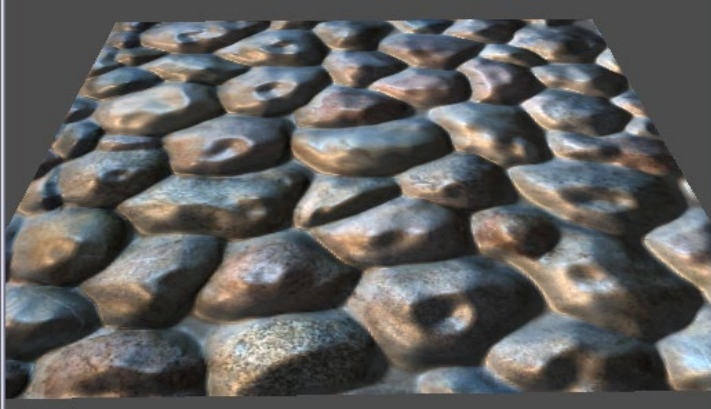




# Textures Beyond Wallpaper: Normal Maps

- from height fields
  - local differences:

$$N(x, y) = \begin{pmatrix} 2\Delta x \\ 0 \\ h(x + \Delta x, y) - h(x - \Delta x, y) \end{pmatrix} \times \begin{pmatrix} 0 \\ 2\Delta y \\ h(x, y + \Delta y) - h(x, y - \Delta y) \end{pmatrix}$$



# Normal Maps

- Multiple Textures


```
gl.activeTexture(gl.TEXTURE0);  
gl.bindTexture(gl.TEXTURE_2D,color);  
  
gl.activeTexture(gl.TEXTURE1);  
gl.bindTexture(gl.TEXTURE_2D,normalmap);
```

- Pixel Shader

```
...  
uniform sampler2D color,normalmap;  
varying vec2 uv; // texture coordinate  
  
void main(void) {  
    ...  
    vec3 c = texture2D(color,uv);  
    vec3 n = texture2D(normalmap,uv);  
    float diff = dot(n,light);  
    ...  
    gl_FragColor = ...  
}
```

# Texture Mapping Demo

Textures



Texture Magnification

☒ NEAREST

☐ LINEAR

Texture Minification

☒ NEAREST - No MIPMap

☐ LINEAR - No MIPMap

☐ NEAREST\_MIPMAP\_NEAREST

☐ LINEAR\_MIPMAP\_NEAREST

☐ NEAREST\_MIPMAP\_LINEAR

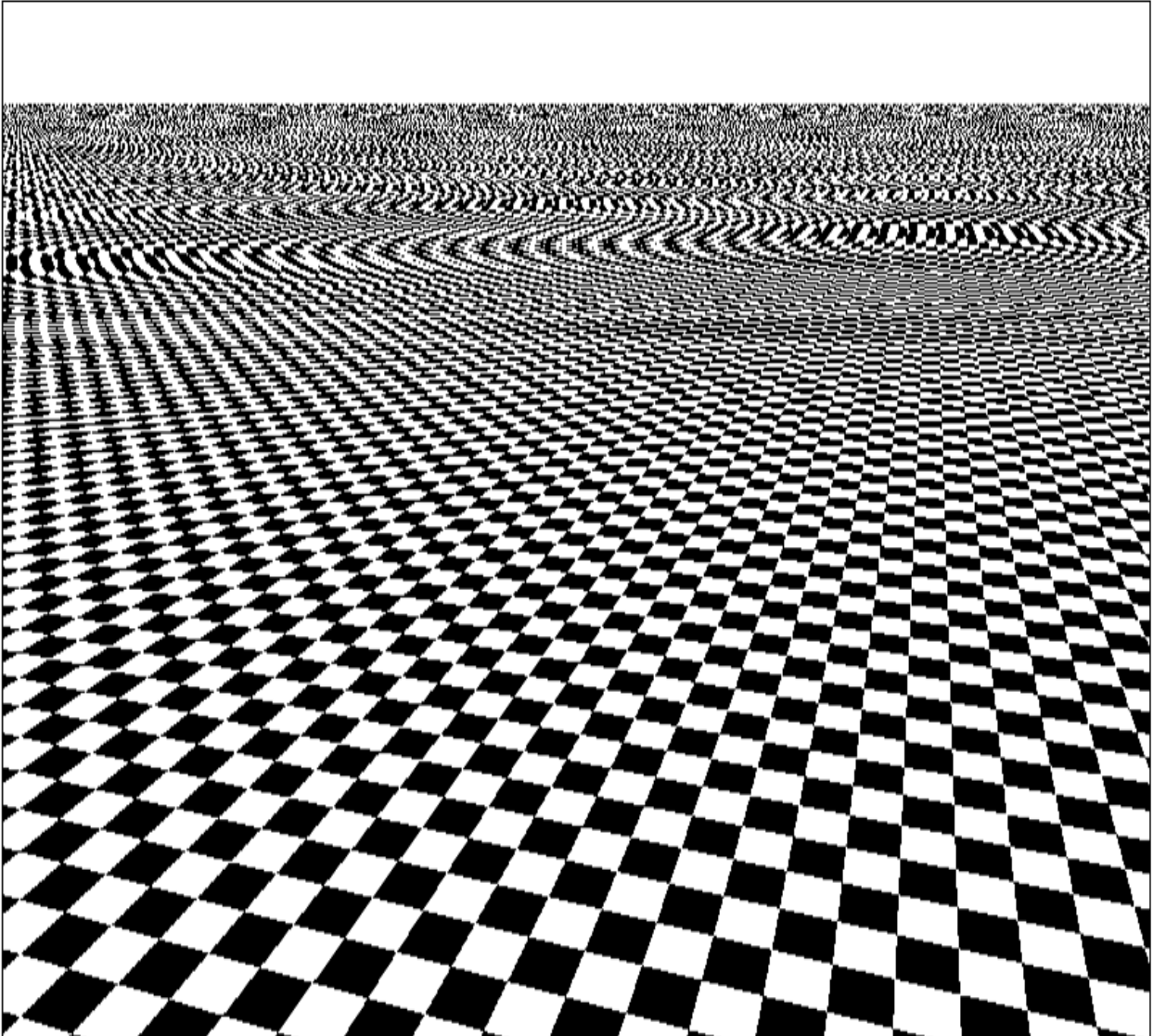
☐ LINEAR\_MIPMAP\_LINEAR

Patch

☒ Infinite Patch

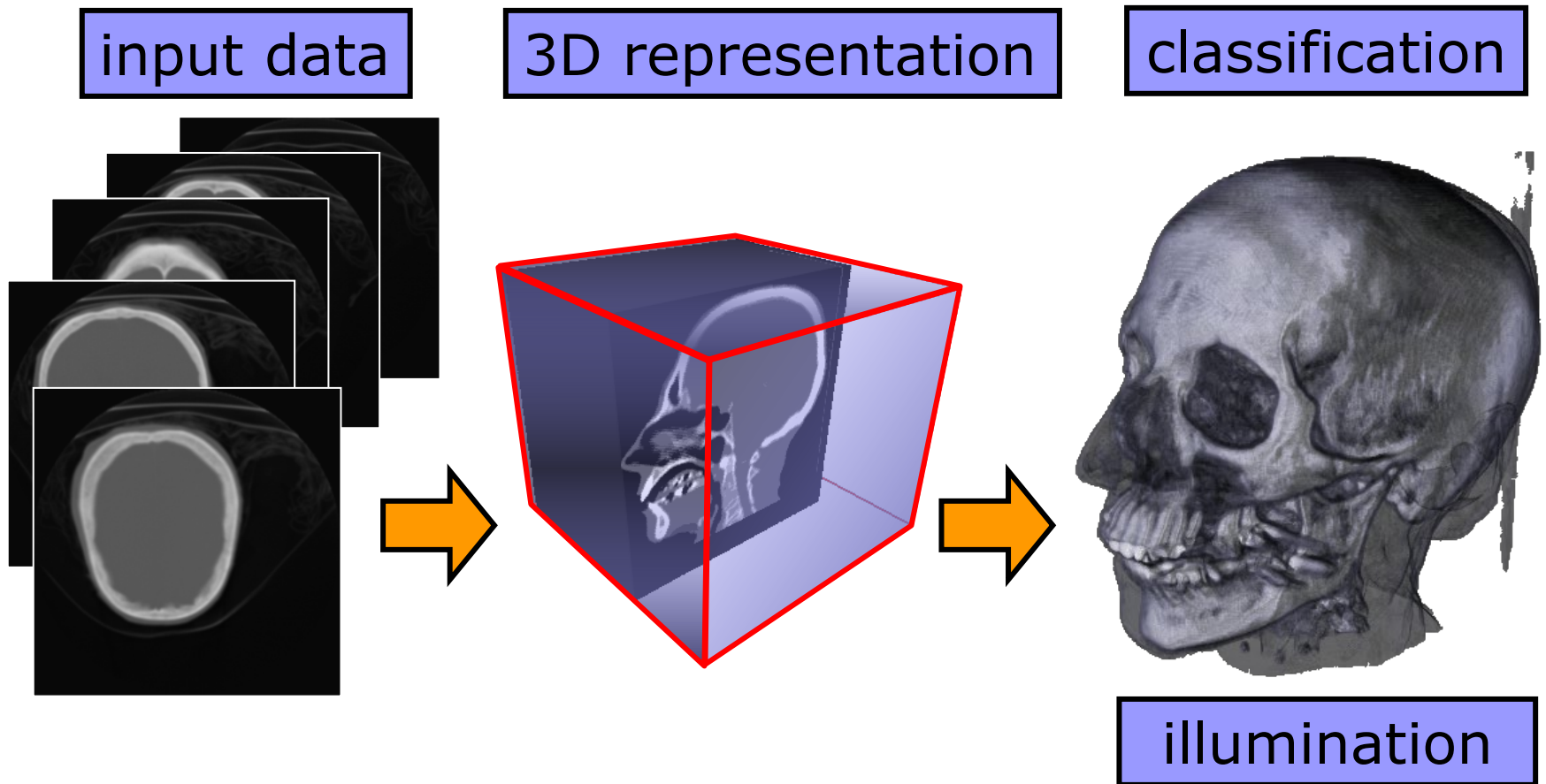
☐ Mirror Patch

☐ Normal Map



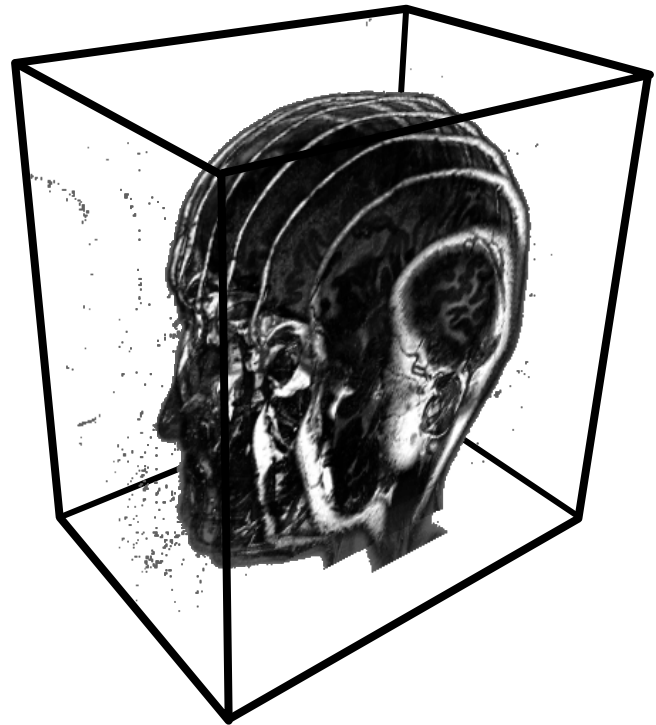
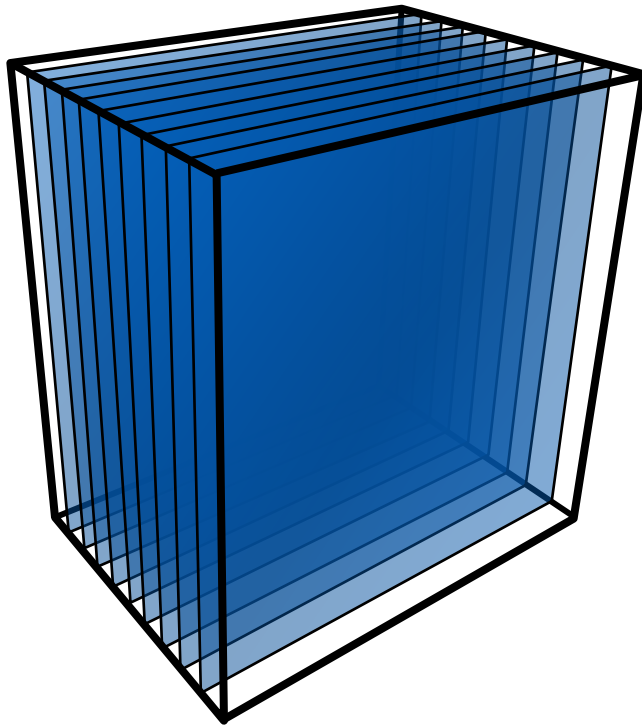
# Volumetric Texture Mapping

- e.g., slices from CT data form a **volumetric texture**



# Volumetric Texture Mapping

- How to render?  
→ Polygonal slices with transparent textures

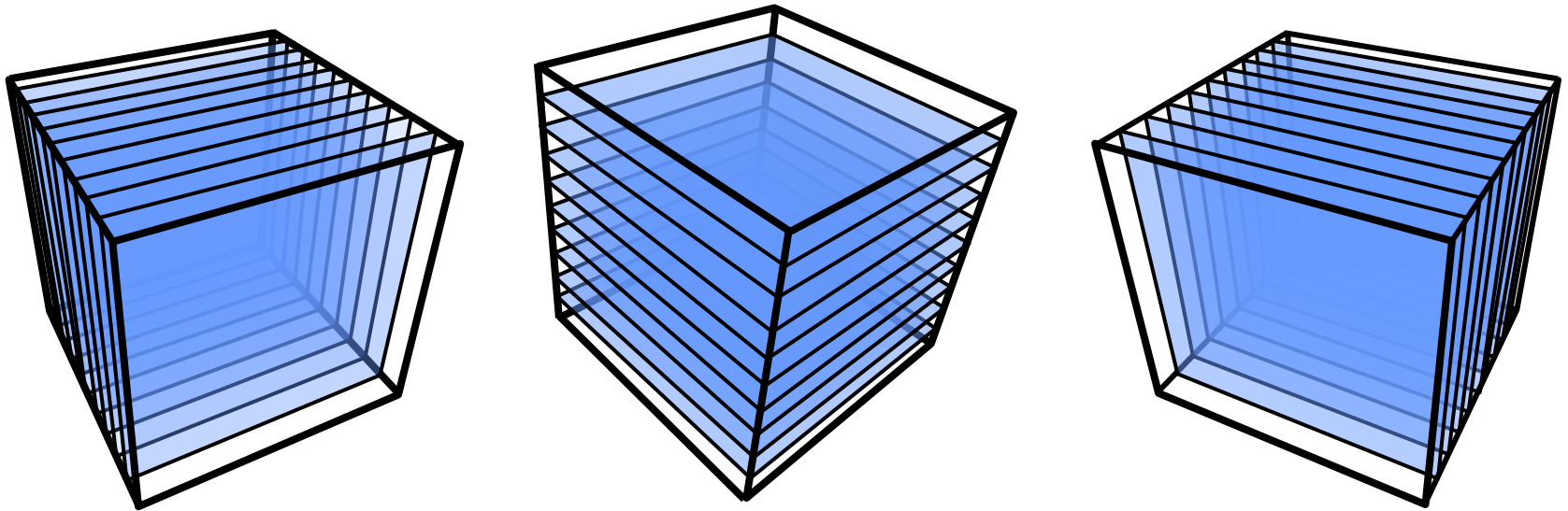


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# Volumetric Texture Mapping

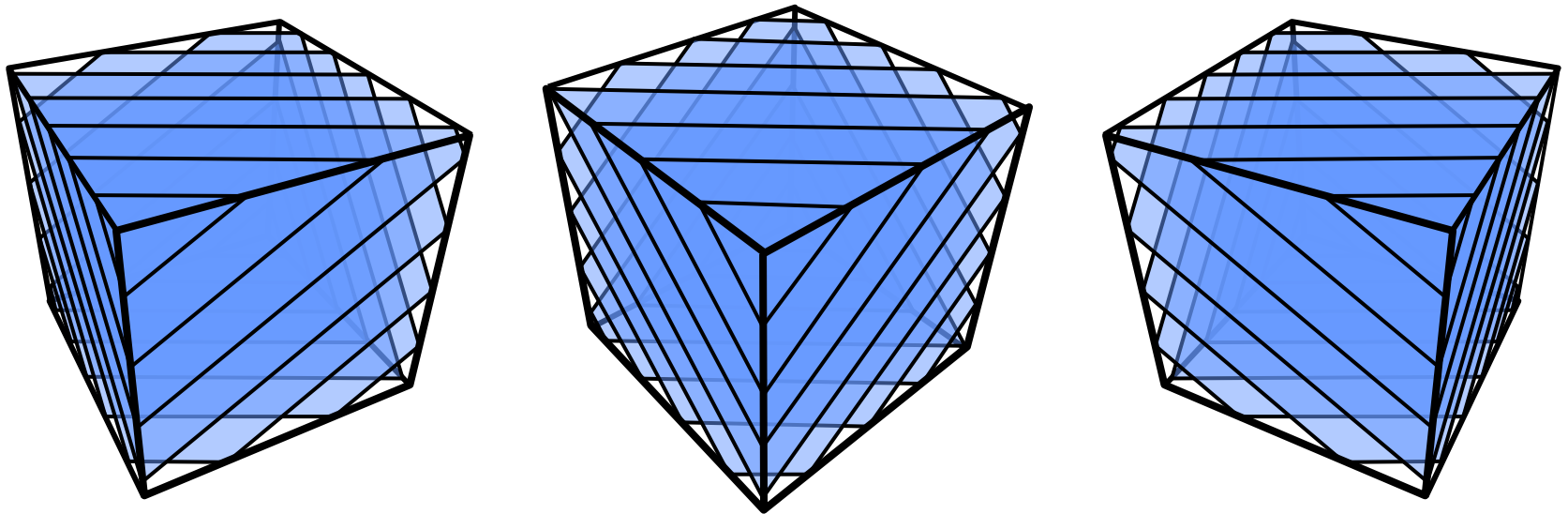
- Variant 1: Axis-aligned slices with 2D textures  
→ 3 copies of the data required



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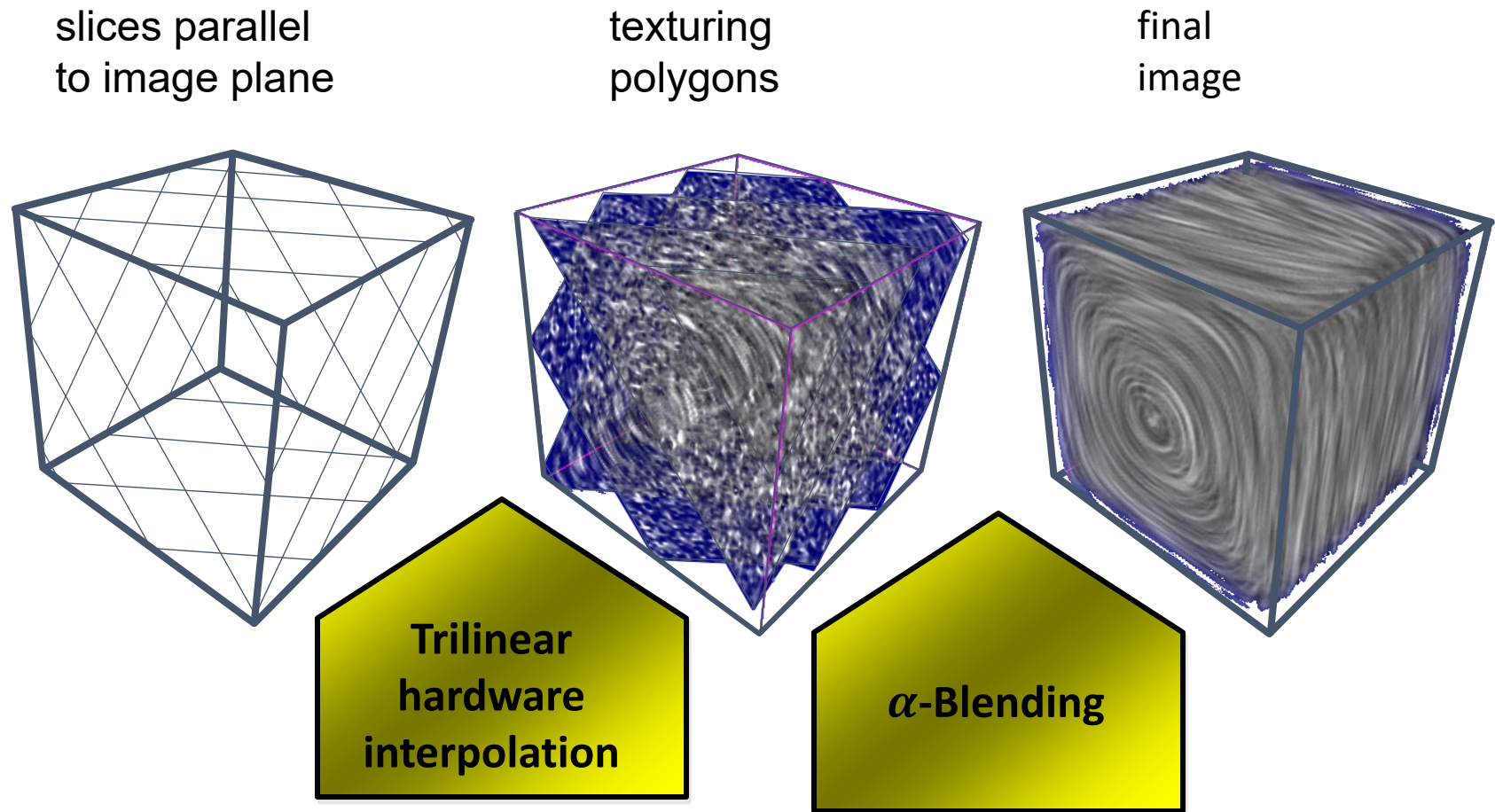
# Volumetric Texture Mapping

- Better: store as 3D texture (supported by OpenGL etc.)  
→ 3D texture coordinates required
- Render slices parallel to image plane back to front  
→ only one copy in texture memory required



Christoph Rezk-Salama

# Volumetric Texture Mapping



Christoph Rezk-Salama

# Next Lecture

- How to interpolate textures
- Texture Aliasing and Antialiasing